ICSE 2025 EXAMINATION

Sample Question Paper - 1

Mathematics

Time Allowed: 2 hours and 30 minutes **Maximum Marks: 80**

General Instructions:

- Answers to this Paper must be written on the paper provided separately.
- You will not be allowed to write during the first 15 minutes.
- This time is to be spent reading the question paper.
- The time given at the head of this Paper is the time allowed for writing the answers.
- **Attempt all questions from Section A and any four questions from Section B**.
- All work, including rough work, must be clearly shown and must be done on the same sheet as the rest of the answers.
- Omission of essential work will result in a loss of marks.
- The intended marks for questions or parts of questions are given in brackets [ ]
- Mathematical tables are provided.

Section A

1. **Question 1 Choose the correct answers to the questions from the given options:** **[15]**

(a) A retailer purchases a fan for ₹1500 from a wholesaler and sells it to a consumer at 10% profit. If the sales are intra-state and the rate of GST is 12%, the cost of the fan to the consumer inclusive of tax is: **[1]**

a) ₹1848 b) ₹1830

c) ₹1650 d) ₹1800

(b) A factory kept increasing its output by the same percentage every year. Then, the percentage, if it is known that the output is doubled in the last two years, will be **[1]**

a) 44.4% b) 14.4%

c) 41.4% d) 44.1%

(c) When $ax^3 + 6x^2 + 4x + 5$ is divided by $(x + 3)$, the remainder is -7. **[1]**
The value of constant a is

a) 2 b) -2

c) -3 d) 3

(d) If $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$, then the value of matrix A^5 is **[1]**

a) $\begin{bmatrix} 87 & 149 \\ 149 & -62 \end{bmatrix}$ b) $\begin{bmatrix} 87 & 149 \\ 149 & 62 \end{bmatrix}$

c) $\begin{bmatrix} 62 & 149 \\ -149 & -87 \end{bmatrix}$

d) $\begin{bmatrix} -62 & -149 \\ 149 & 87 \end{bmatrix}$

(e) An AP starts with a positive fraction and every alternate term is an integer. If the sum of the first 11 terms is 33, then the fourth term is **[1]**

a) 3

b) 6

c) 5

d) 2

(f) If (4, 3) and (-4, -3) are opposite two vertices of a rectangle, then other two vertices are **[1]**

a) (4, -3) and (-4, 3)

b) (-4, -3) and (-4, -3)

c) (-4, 4) and (-3, 4)

d) (4, -3) and (-3, 4)

(g) Through the mid-point M of the side CD of a parallelogram ABCD, the line BM is drawn intersecting AC at L and AD produced at E. The values of EL and ar (△AEL) are respectively **[1]**

a) ar (△CBL) and BL

b) 2BL and 4 ar (△CBL)

c) 4 ar (△CBL) and 2BL

d) BL and ar (△CBL)

(h) A sphere of radius a units is immersed completely in water contained in a right circular cone of semi-vertical angle 30° and water is drained off from the cone till its surface touches the sphere. Then, the volume of water remaining in the cone will be **[1]**

a) $\frac{5}{3}\pi a^2$

b) $\frac{5\pi}{3}a^3$

c) $\frac{\pi a^3}{3}$

d) $5\pi a^3$

(i) Graph the range of the inequation $-2\frac{2}{3} \leq x + \frac{1}{3} \leq 3\frac{1}{3}, \forall x \in R$ on the number line. If the solution set is consider as a diagonal of a square on the number line, then the area of obtained figure, is **[1]**

a) 11 sq units

b) 14 sq units

c) 17 sq units

d) 18 sq units

(j) The probability that the minute hand lies from 5 to 15 min in the wall clock, is **[1]**

a) $\frac{1}{6}$

b) $\frac{5}{6}$

c) $\frac{1}{5}$

d) $\frac{1}{10}$

(k) If $A = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$, then A^n (where, n is a natural number) is equal to **[1]**

a) $\begin{bmatrix} 3n & 0 \\ 0 & 3n \end{bmatrix}$

b) $3^n \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

c) $\begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$

d) $I_{2 \times 2}$

(l) The sum of the squares of the distances of a moving point (x, y) from two fixed points (a, 0) and (- a, 0) is equal to a constant quantity $2b^2$. The value of $x^2 + y^2 + a^2$ is equal to **[1]**

a) b^2

b) $-a^2$

c) ab

d) $-b^2$

(m) If P, Q, S and R are points on the circumference of a circle of radius r, such that PQR is an equilateral triangle and PS is a diameter of the circle. Then, the perimeter of the quadrilateral PQSR will be **[1]**

a) $2(\sqrt{3} + 1)r$ b) $2\sqrt{3} + r$

c) 2r d) $2\sqrt{3}r$

(n) Observe the data given in three sets **[1]**

P: 3, 5, 9, 12, x, 7, 2

Q: 8, 2, 1, 5, 7, 9, 3

R: 5, 9, 8, 3, 2, 7, 1

If the ratio between P's and Q's means is 7 : 5, then the ratio between P's and R's means is

a) 7 : 5 b) 5 : 7

c) 6 : 7 d) 7 : 6

(o) **Assertion (A):** Sum of first 10 terms of the arithmetic progression -0.5, -1.0, -1.5, ... is 27.5 **[1]**

Reason (R): Sum of n terms of an A.P. is given as $S_n = \frac{n}{2}[2a + (n-1)d]$ where a = first term, d = common difference.

a) Both A and R are true and R is the correct explanation of A. b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false. d) A is false but R is true.

2. **Question 2** **[12]**

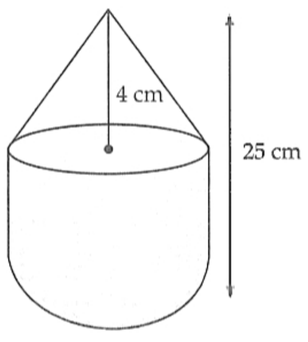
(a) Mrs. chopra deposits ₹1600 per month in a Recurring Deposit Account at 9% per annum simple interest. If she gets ₹65592 at the time of maturity, then find the total time for which the account was held. **[4]**

(b) Find the mean proportional of $(a^4 - b^4)^2$ and $[(a^2 - b^2)(a - b)]^{-2}$. **[4]**

(c) If cosec $\theta = x + \frac{1}{4x}$, then prove that cosec θ + cot $\theta = 2x$ or $\frac{1}{2x}$. **[4]**

3. **Question 3** **[13]**

(a) The given solid figure is cylinder surmounted by a cone. The diameter of the base of the cylinder is 6 cm. The height of the cone is 4 cm and the total height of the solid is 25 cm. Take $\pi = \frac{22}{7}$. **[4]**



Find the:

i. Volume of the solid
ii. Curved surface area of the solid

Give your answer correct to the nearest whole number.

(b) The equation of a line is y = 3x - 5. Write down the slope of this line and the intercept made by its on the Y-axis. Hence or otherwise, write down the equation of a line, which is parallel to the line and which passes through the point (0, 5). **[4]**

(c) Use graph paper for this question (Take 2 cm = 1 unit along both x and y axis). ABCD is a quadrilateral whose vertices are A(2, 2), B(2, - 2), C (0, -1) and D (0, 1) **[5]**

i. Reflect quadrilateral ABCD on the y-axis and name it as A'B'CD.

ii. Write down the coordinates of A' and B'

iii. Name two points which are invariant under the above reflection.

iv. Name the polygon A'B'CD.

Section B

Attempt any 4 questions

4. **Question 4** **[10]**

(a) The price of a Barbie Doll is ₹ 3136 inclusive tax (under GST) at the rate of 12% on its listed price. A buyer asks for a discount on the listed price, so that after charging GST, the selling price becomes equal to the listed price. Find the amount of discount which the seller has to allow for the deal. **[3]**

(b) Find the values of k, for which the equation $x^2 + 5kx + 16 = 0$ has no real roots. **[3]**

(c) The mean of the following distribution is 49. Find the missing frequency **a**. **[4]**

Class Interval	0-20	20-40	40-60	60-80	80 -100
Frequency	15	20	30	a	10

5. **Question 5** **[10]**

(a) Find the values of x, y, a and b, when $\begin{bmatrix} x+y & a-b \\ a+b & 2x-3y \end{bmatrix} = \begin{bmatrix} 5 & 3 \\ -1 & -5 \end{bmatrix}$. **[3]**

(b) Two chords AB and CD of a circle intersect each other at a point E inside the circle. If AB = 9 cm, AE = 4 cm and ED = 6 cm, then find CE. **[3]**

(c) Determine, whether the polynomial g(x) = x - 7 is a factor of $f(x) = x^3 - 6x^2 - 19x + 84$ or not. **[4]**

6. **Question 6** **[10]**

(a) Find the points of trisection of the line segment joining the points (5, -6) and (-7, 5). **[3]**

(b) Prove the following identities. **[3]**

i. $\sin^4\theta + \cos^4\theta = 1 - 2\sin^2\theta\cos^2\theta$

ii. $\frac{1}{\operatorname{cosec}\theta - \cot\theta} - \frac{1}{\sin\theta} = \frac{1}{\sin\theta} - \frac{1}{\operatorname{cosec}\theta + \cot\theta}$

(c) 150 workers were engaged to finish a job in a certain number of days, 4 workers dropped out on second day, 4 more workers dropped out an third day and so on. It took 8 more days of finish the work. Find the number of days in which the work was completed. **[4]**

7. **Question 7** **[10]**

(a) A two-digit positive number, such that the product of its digits is 6. If 9 is added to the number, then the digits interchange their places. Find the number. **[5]**

(b) The marks obtained by 120 students in a test are given below: **[5]**

Marks	Number of Students
0 - 10	5
10 - 20	9
20 - 30	16
30 - 40	22
40 - 50	26
50 - 60	18

60 - 70	11
70 - 80	6
80 - 90	4
90 - 100	3

Draw an ogive for the given distribution on a graph sheet.

(Use suitable scale for ogive to estimate the following)

i. the median.

ii. the number of students who obtained more than 75% marks in the test.

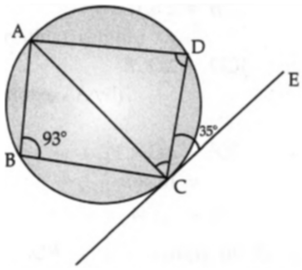
iii. the number of students who did not pass the test, if minimum marks required to pass is 40.

8. **Question 8** **[10]**

(a) Two players Niharika and Shreya play a tennis match. It is known that the probability of Niharika winning the match is 0.62. What is the probability of Shreya winning the match? **[3]**

(b) A conical military tent is 5 m high and the diameter of the base is 24 m. Find the cost of canvas used in making this tent at the rate of ₹ 14 per sq m. **[3]**

(c) In the given figure CE is a tangent to the circle at point C. ABCD is a cyclic quadrilateral. If $\angle$ABC = 93^o and $\angle$DCE = 35^o **[4]**



find:

i. $\angle ADC$

ii. $\angle CAD$

iii. $\angle ACD$

9. **Question 9** **[10]**

(a) Given: A = {x : 3 < 2x - 1 < 9, x $\in$ R}, B = {x : 11 $\leq$ 3x + 2 $\leq$ 23, x $\in$ R} where R is the set of real number. **[3]**

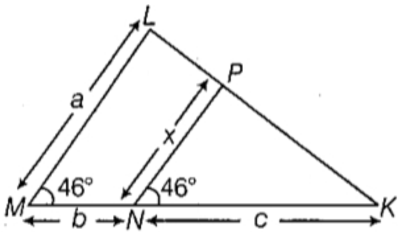
i. Represents A and B on number lines

ii. On the number line also mark A $\cap$ B.

(b) Find the missing frequency for the given frequency distribution table, if the mean, of the distribution is 18. **[3]**

Class interval	11-13	13-15	15-17	17-19	19-21	21-23	23-25
Frequency	3	6	9	13	f	5	4

(c) In the given figure, $\angle$M = $\angle$N = 46^o. Express x in terms of a, b and c, where a, b and c are the lengths of LM, MN and NK, respectively. **[4]**

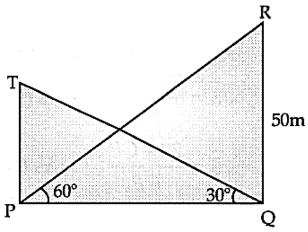


10. **Question 10** **[10]**

(a) The ages of A and B are in the ratio 7 : 8. Six years ago, their ages were in the ratio 5 : 6. Find their present ages. **[3]**

(b) Draw a circle of radius 6 cm. From a point 10 cm away from its centre, construct the pair of tangents to the circle and measure their lengths. **[3]**

(c) The angle of elevation from a point P of the top of a tower QR, 50 m high is 60^{o} and that the tower PT from a point Q is 30^{o}. Find the height to the tower PT, correct to the nearest metre. **[4]**



Solution

Section A

1. Question 1 Choose the correct answers to the questions from the given options:

(i) **(a)** ₹1848

Explanation: {

Here, selling price of fan = ₹1650

GST on fan = 12% of ₹ 1650

= 1650 $\times \frac{12}{100}$

= 198

Thus, cost of a fan to the consumer inclusive of tax

= ₹(1650 + 198) = ₹1848

(ii) **(c)** 41.4%

Explanation: {

Let P be the initial production (2 yr ago) and the increase in production every year be x%. Then, production at the end of first year = P + $\frac{P_X}{100}$ = $P\left(1+\frac{x}{100}\right)$

Production at the end of second year

= P = $\left(1+\frac{x}{100}\right)$ + $\frac{Px}{100}\left[\left(1+\frac{x}{100}\right)\right]$

= P $\left(1+\frac{x}{100}\right)\left(1+\frac{x}{100}\right)$ = P$\left(1+\frac{x}{100}\right)^2$

Since, the production is doubled in last two years.

$\therefore$ P$\left(1+\frac{x}{100}\right)^2$ = 2P $\Rightarrow$ $\left(1+\frac{x}{100}\right)^2$ = 2

$\Rightarrow$ (100 + x)^2 = 2 $\times$ (100)^2 $\Rightarrow$ 10000 + x^2 + 200x = 20000

$\Rightarrow$ On comparing it with ax^2 + bx + c = 0, we get

a = 1, b = 200 and c = -10000

By quadratic formula, x = $\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

$\therefore$ x = $\frac{-200\pm\sqrt{(200)^2+40000}}{2}$

= -100 $\pm$ 100$\sqrt{2}$ = 100(-1 $\pm$ 4$\sqrt{2}$)

= 100 (-1 + 1.414) [$\because$ percentage cannot be negative]

= 100(0.414) = 41.4

Hence, the required percentage is 41.4%.

(iii) **(a)** 2

Explanation: {

Let f(x) = ax^3 + 6x^2 + 4x + 5

By remainder theorem, f(-3) = -7

$\Rightarrow$ a(-3)^3 + 6(-3)^2 + 4(-3) + 5 = -7

$\Rightarrow$ -27a + 54 - 12 + 5 = -7

$\Rightarrow$ -27a = -54

$\Rightarrow$ a = 2

(iv) **(c)** $\begin{bmatrix} 62 & 149 \\ -149 & -87 \end{bmatrix}$

Explanation: {

Given, $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$

Now, A^2 = A $\cdot$ A = $\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$

$= \begin{bmatrix} 9-1 & 3+2 \\ -3-2 & -1+4 \end{bmatrix} = \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix}$

$\therefore$ $A^4 = A^2 \cdot A^2 = \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix} \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix} = \begin{bmatrix} 64-25 & 40+15 \\ -40-15 & -25+9 \end{bmatrix}$

$= \begin{bmatrix} 39 & 55 \\ -55 & -16 \end{bmatrix}$

Now, $A^5 = A^4 \cdot A = \begin{bmatrix} 39 & 55 \\ -55 & -16 \end{bmatrix} \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$

$= \begin{bmatrix} 117-55 & 39+110 \\ -165+16 & -55-32 \end{bmatrix} = \begin{bmatrix} 62 & 149 \\ -149 & -87 \end{bmatrix}$

(v) **(d)** 2

Explanation: {

Given, $S_{11} = 33$

$\Rightarrow \frac{11}{2}$ (2a + 10d) = 33 [$\because S_n = \frac{n}{2}$ [2a + (n - 1)d]

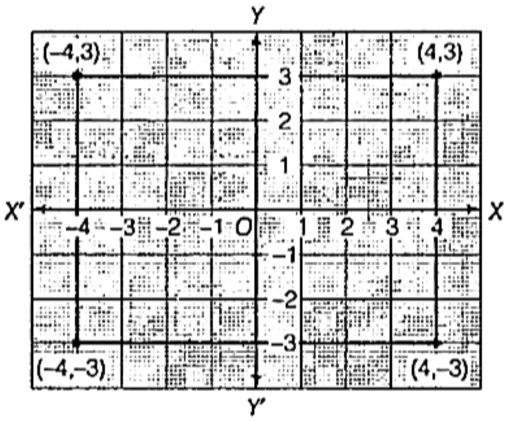
$\Rightarrow$ a + 5d = 3

i.e. $a_6 = 3 \Rightarrow a_4 = 2$ [$\because$ alternate terms are integers and the given sum is possible]

(vi) **(a)** (4, -3) and (-4, 3)

Explanation: {

Since, the image of point (4, 3) under X-axis is (4, - 3) and the image of point (4, 3) under Y-axis is (-4, 3).

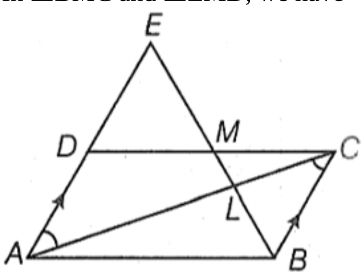


$\therefore$ Other two vertices of the rectangle are (4, -3) and (-4, 3).

(vii) **(b)** 2BL and 4 ar ($\triangle$CBL)

Explanation: {

In $\triangle$BMC and $\triangle$EMD, we have



$\angle$BMC = $\angle$EMD [vertically opposite angles]

$\Rightarrow$ MC = MD [$\because$ M is the mid-point of CD]

$\Rightarrow$ $\angle$MCB = $\angle$MDE [alternate angles]

So, by AAS congruence criterion, we have

$\triangle BMC \cong \triangle EMD$

$\Rightarrow$ BC = ED [$\because$ corresponding parts of congruent triangles are equal]

In $\triangle$AEL and $\triangle$CBL, we have

$\angle$ALE = $\angle$CLB [vertically opposite angles]

and $\angle$EAL = $\angle$BCL [alternate angles]

So, by AA criterion of similarity, we have

$\triangle AEL \sim \triangle CBL$

$\Rightarrow \frac{AE}{BC} = \frac{EL}{BL} = \frac{AL}{CL}$ [$\because$ if two triangles are similar, then their corresponding sides are proportional]

On taking first two terms, we get

$\frac{EL}{BL}=\frac{AE}{BC}=\frac{AD+DE}{BC}$

$=\frac{BC+BC}{BC}=\frac{2BC}{BC}$ = 2 [$\because$ AD = SC as sides opposite to parallelogram and DE = BC, proved above]

$\Rightarrow$ EL = 2BL ...(i)

Now, $\frac{\operatorname{ar}(\Delta AEL)}{\operatorname{ar}(\Delta CBL)}=\left(\frac{EL}{BL}\right)^2$ [$\because$ ratio of areas of two similar triangles is equal to the square of the ratio of their corresponding sides]

$=\left(\frac{2BL}{BL}\right)^2$ = $(2)^2$ [from Eq. (i)]

$\Rightarrow \frac{\operatorname{ar}(\triangle AEL)}{\operatorname{ar}(\triangle CBL)}$ = 4

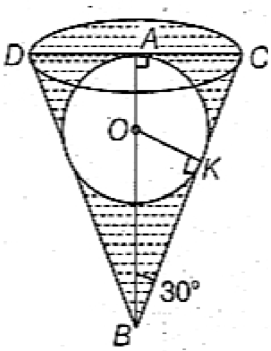
$\Rightarrow$ ar ($\triangle$AEL) = 4 ar ($\triangle$CBL)

(viii) **(b)** $\frac{5\pi}{3}a^3$

Explanation: {

Let radius of sphere be a, i.e. OK = OA = a.

Then, the centre O of a sphere will be centroid of the $\triangle$BCD



$\therefore$ OA = $\frac{1}{3}$AB $\Rightarrow$ AB = 3(OA)

In right angled $\triangle$OKB,

sin 30^o = $\frac{OK}{OB}$ = $\frac{a}{OB}$

$\Rightarrow \frac{1}{2}$ = $\frac{a}{OB}$

$\Rightarrow$ OB = 2a

Now, AB = OA + OB = a + 2a = 3a

Now, in right angled $\triangle$BAC,

$\frac{AC}{AB}$ = tan 30^o $\Rightarrow \frac{AC}{AB}$ = $\frac{1}{\sqrt{3}}$

$\Rightarrow$ AC = $\frac{AB}{\sqrt{3}}$ = $\frac{3a}{\sqrt{3}}$

$\therefore$ AC = $\sqrt{3a}$ units

Now, volume of a cone BCD = $\frac{1}{3}\pi$ $(AC)^2 \times$ AB

= $\frac{1}{3}\pi(a\sqrt{3})^2\times 3a$ = $3\pi a^3$

$\therefore$ Volume of water remaining in the cone = Volume of the cone BCD - Volume of a sphere

= $3\pi a^3$ - $\frac{4}{3}\pi a^3$ = $\frac{5\pi}{3}a^3$ cu units

(ix) **(d)** 18 sq units

Explanation: {

Given, $-2\frac{2}{3}\leq x+\frac{1}{3}\leq 3\frac{1}{3}$

$\Rightarrow \frac{-8}{3}\leq x+\frac{1}{3}\leq\frac{10}{3}$

$\frac{-8}{3}\times 3\leq\left(x+\frac{1}{3}\right)3\leq\frac{10}{3}\times 3$ [multiplying by 3 in each term]

$\Rightarrow$ -8 $\leq$ 3x + 1 $\leq$ 10

$\Rightarrow$ -8 - 1 $\leq$ 3x + 1 - 1 $\leq$ 10 - 1 [subtracting 1 from each term]

$\Rightarrow$ -9 $\leq$ 3 x $\leq$ 9

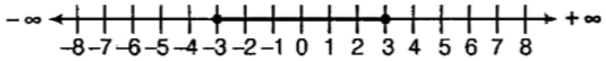
$\Rightarrow \frac{-9}{3}\leq\frac{3x}{3}\leq\frac{9}{3}$ [dividing by 3 each term]

$\Rightarrow$ -3 $\leq$ x $\leq$ 3

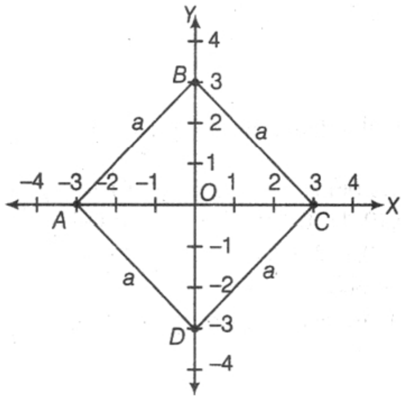
Since, x $\in$ R.

$\therefore$ Range of x is [-3, 3].

Representation of range of x on the number line is given as



Now, consider the following figure



Here, AC = 6 units, which is a diagonal of square.

Let side of a square ABCD be a.

In right angled $\triangle$ABC,

$AC^2 = AB^2 + BC^2$

$\Rightarrow 6^2 = a^2 + a^2$

$\Rightarrow 36 = 2a^2$

$\Rightarrow a^2 = 18$

Now, area of a square ABCD= $(\text{Side})^2 = a^2 = 18$ sq units.

(x) **(a)** $\frac{1}{6}$

Explanation: {

In a wall clock, the minute hand cover the 60 min in on complete round.

$\therefore$ Total number of possible outcomes = 60

The minute hand cover the time from 5 to 15 min,

Number of outcomes favourable to E

= Distance from 5 to 15 min = 10

$\therefore$ Required probability = $\frac{10}{60} = \frac{1}{6}$

(xi) **(b)** $3^n \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Explanation: {

We have, $A = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix} = 3 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ = 3l

$\therefore A^n = (3l)^n = 3^n l^n = 3^n l$ [$\because l^n = l$, for all natural numbers n]

$= 3^n \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

(xii) **(a)** b^2

Explanation: {

Let P (x, y) be the moving point.

Let given two fixed points be A (a, 0) and B (-a, 0).

According to the given condition,

$PA^2 + PB^2 = 2b^2$

$\Rightarrow (x - a)^2 + (y - 0)^2 + (x + a)^2 + (y - 0)^2 = 2b^2$ [by distance formula]

$\Rightarrow x^2 - 2ax + a^2 + y^2 + x^2 + 2ax + a^2 + y^2 = 2b^2$

$\Rightarrow 2x^2 + 2y^2 + 2a^2 = 2b^2$

$\Rightarrow x^2 + y^2 + a^2 = b^2$ [dividing both sides by 2]

(xiii) **(a)** $2(\sqrt{3} + 1)r$

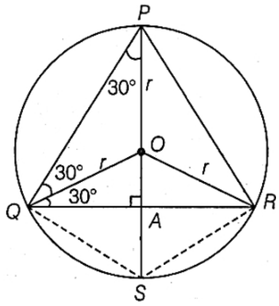
Explanation: {

As PQR is an equilateral triangle, hence PS will be perpendicular to QP and will divide it into 2 equal parts.

Since, $\angle$P and $\angle$S will be supplementary, so

$\angle S = 120^o$ and $\angle QSA = \angle RSA = 60^o$

Now, PA = PQ cos 30^o and OA = OQ sin 30^o = $\frac{r}{2}$



$\Rightarrow$ AS = OA = $\frac{r}{2}$ and PA = PO + OA = r + $\frac{r}{2}$

Hence, PQ = $\frac{PA}{\cos 30^\circ} = \frac{r+\frac{r}{2}}{\frac{\sqrt{3}}{2}} = \sqrt{3}r$

In $\triangle$QAS, AS = QS cos 60^o $\Rightarrow$ QS = $\frac{\frac{r}{2}}{\frac{1}{2}}$ = r

Since, AQ = AR, AS is common and $\angle QAS = \angle RAS = 90^o$

So, QS = RS.

$\therefore$ Perimeter of PQSP = 2(PQ + QS) = 2($\sqrt{3}$ + 1)r

(xiv) **(a)** 7 : 5

Explanation: {

Mean of the observations of sets

P = $\frac{3+5+9+12+x+7+2}{7} = \frac{38+x}{7}$

Q = $\frac{8+2+1+5+7+9+3}{7} = \frac{35}{7}$ = 5

and R = $\frac{5+9+8+3+2+7+1}{7} = \frac{35}{7}$ = 5

$\therefore$ Ratio of means of sets P and Q = 7 : 5 [given]

Let P's mean = 7y and Q's mean = 5y.

$\Rightarrow$ 5y = 5

$\Rightarrow$ y = 1

$\therefore$ Q's mean = 5

Now, P's mean = 7y

$\Rightarrow$ $\frac{38+x}{7}$ = 7 $\times$ 1 $\Rightarrow$ 38 + x = 48 $\Rightarrow$ x = 11

$\therefore$ Mean of P : Mean of R = 7y : 5 = 7 $\times$ 1 : 5 = 7 : 5

(xv) **(a)** Both A and R are true and R is the correct explanation of A.

Explanation: {

Both are correct. Reason is the correct reasoning for Assertion.

Assertion, S_{10} = $\frac{10}{2}$[2(-0.5) + (10 - 1)(-0.5)]

= 5[-1 - 4.5]

= 5(-5.5) = 27.5

2. Question 2

(i) p = ₹ 1600/month

r = 9% p.a.

m.v. = 65,592

n = ?

m.v. = $pn + \frac{\text{p}\cdot\text{r}\cdot\text{n}(n+1)}{2400}$

$\Rightarrow$ 65,592 = $1600n + \frac{1600\times 9\times n(n+1)}{2400}$

$\Rightarrow$ 65,592 = 1600n + 6n(n + 1)

$\Rightarrow$ 65,592 = 1600n + $6n^2$ + 6n

$\Rightarrow$ $6n^2$ + 1606n - 65,592 = 0

$n = \frac{-1606 \pm \sqrt{(1606)^2 - 4(6)(-65,592)}}{2(6)}$

$n = \frac{-1606 \pm \sqrt{4153444}}{12}$

$n = \frac{-1606 \pm 2038}{12}$

$n = \frac{-1606 + 2038}{12}$

$n = \frac{432}{12}$

n = 36 months

n = 3 years

or $n = \frac{-1606 - 2038}{12}$

or $n = \frac{-3649}{12}$

or n = -303.66 months rejected.

As 'n' is no. of months here. So can't be -ve.

(ii) Let the mean proportional between $(a^4 - b^4)^2$ and $[(a^2 - b^2)(a - b)]^{-2}$ be x.

$\Rightarrow (a^4 - b^4)^2$, x and $[(a^2 - b^2)(a - b)]^{-2}$ are in continued proportion.

$\Rightarrow (a^4 - b4)^2 : x = x : [(a^2 - b^2)(a - b)]^{-2}$

$x^2 = (a^4 - b^4)^2 \cdot [(a^2 - b^2)(a - b)]^{-2}$

$\Rightarrow x^2 = \frac{\left(a^4 - b^4\right)^2}{\left[\left(a^2 - b^2\right)(a - b)\right]^2}$

$\Rightarrow x = \frac{a^4 - b^4}{\left(a^2 - b^2\right)(a - b)}$

$\Rightarrow x = \frac{\left(a^2 + b^2\right)\left(a^2 - b^2\right)}{\left(a^2 - b^2\right)(a - b)}$

$\Rightarrow x = \frac{a^2 + b^2}{a - b}$

(iii) Given, cosec θ = x + $\frac{1}{4x}$ ...(i)

We know that, $\cot^2 \theta = \operatorname{cosec}^2 \theta - 1$

$\Rightarrow \cot^2 \theta = \left(x + \frac{1}{4x}\right)^2 - 1$ [from Eq. (i)]

$\Rightarrow \cot^2 \theta = x^2 + \frac{1}{16x^2} + 2x \cdot \frac{1}{4x} - 1$ [$\because (a + b)^2 = a^2 + b^2 + 2ab$]

$= x^2 + \frac{1}{16x^2} + \frac{1}{2} - 1 = x^2 + \frac{1}{16x^2} - \frac{1}{2}$

$= x^2 + \frac{1}{16x^2} - 2x \cdot \frac{1}{4x} = \left(x - \frac{1}{4x}\right)^2$ [$\because a^2 + b^2 - 2ab = (a - b)^2$]

$\Rightarrow \cot \theta = x - \frac{1}{4x}$ ...(ii)

or $\cot \theta = -\left(x - \frac{1}{4x}\right)$ ...(iii)

On adding Eqs. (i) and (ii), we get

cosec θ + cot θ = 2x

Now, adding Eqs. (i) and (iii), we get

cosec θ + cot θ = $\frac{1}{2x}$

Hence, cosec θ + cot θ = 2x or $\frac{1}{2x}$.

3. Question 3

(i) Given total height of the solid = 25 cm

Height of the cone (h_2) = 4 cm

Diameter of the cylinder = 6 cm

Height of the cylinder (h_1) = 25 - 4 = 21 cm

Radius of the cone = Radius of the cylinder = (r)

= $\frac{6}{2}$ = 3 cm

Slant height of cone = $\sqrt{h_2^2 + r^2}$

$= \sqrt{4^2 + 3^2} = \sqrt{16 + 9}$

$= \sqrt{25}$

= 5 cm.

i. Volume of the solid = Volume of cylinder + Volume of cone

$= \pi r^2 h_1 + \frac{1}{3}\pi r^2 h_2$

$= \pi r^2 \left(h_1 + \frac{1}{3}h_2\right)$

$= \frac{22}{7} \times 3 \times 3 \times \left(21 + \frac{4}{3}\right)$

$= \frac{22}{7} \times 9 \times \frac{67}{3}$

= 631.71 $cm^3 \approx$ 632 cm^3. (Approx.)

ii. Curved surface area of the solid = C.S.A of cylinder + C.S.A. of cone

$= 2\pi r h_1 + \pi r l$

$= \pi r(2h_1 + l)$

$= \frac{22}{7} \times 3(2 \times 21 + 5)$

$= \frac{22}{7} \times 3 \times 47 = 443.14$ cm^2

Curved surface area = 443 cm^2 (Approx.).

(ii) Given eqn of line y = 3x - 5

Comprare with y = mx + c we get.

Slope (m) = 3 and

y-intercept (c) = -5

Now slope of the line parellel to the given line will be 3 and it passes through (0, 5).

Thus eqn of line will be

$y - y_1 = m(x - x_1)$

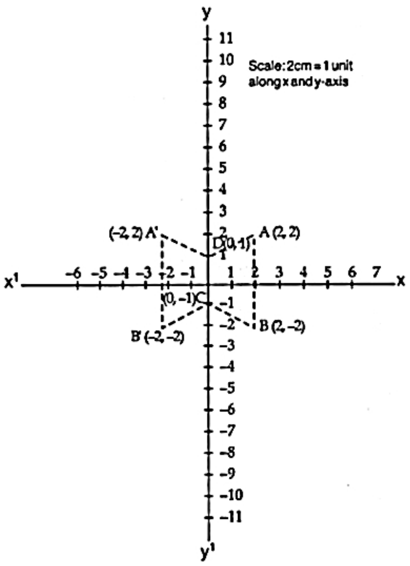
y - 5 = 3(x - 0)

y - 5 = 3x

Y= 3x + 5

(iii)

i.



ii. Co-ordinates of A' $\rightarrow$ (-2, 2)

Co-ordinates of B' $\rightarrow$ (-2, - 2)

iii. Two invariant points are C (0, -1) and D (0,1).

iv. A'B'CD is an isosceles Trapezium polygon.

Section B

4. Question 4

(i) Let the List price of doll be ₹ x.

Total Amount = x + 12% of x

$= x + \frac{12}{100}x$

$= \frac{112}{100}x$

ATQ $\frac{112}{100}x = 3136$

$x = \frac{3136 \times 100}{112}$

x = 2800

$\therefore$ List price of doll ₹ 2800.

Now the reduced price of the doll

= ₹(2800 - y)

amount of GST on ₹ (2800 - y)

12% of (2800 - y)

= ₹ $\frac{12}{100}$ (2800 - y)

Now the selling price of doll

= (2800 - y) + $\frac{12}{100}(2800-y)$

= (2800 - y)$\left(1+\frac{12}{100}\right)$

= (2800 - y) $\frac{112}{100}$

According to given condition, selling price of doll = list price of doll (i.e 2800)

i.e $\frac{112}{100}(2800-y)$ = 2800

2800 - y = $\frac{2800\times100}{112}$

2800 - y = 2500

2800 - 2500 = y

y = 300

Hence, amount of discount is ₹ 300.

(ii) Given equation is $x^2 + 5kx + 16 = 0$

On comparing it with $ax^2 + bx + c = 0$, we get

a = 1, b = 5k and c = 16

Now, discriminant, D = b^2 - 4ac

= $(5k)^2 - 4 \times 1 \times 16 = 25k^2 - 64$

Since, the given equation has no real roots.

$\therefore$ D < 0

$\Rightarrow 25k^2 - 64 < 0$

$\Rightarrow 25\left(k^2-\frac{64}{25}\right) < 0 \Rightarrow k^2-\frac{64}{25} < 0$

$\Rightarrow k^2 < \frac{64}{25} \Rightarrow -\frac{8}{5} < k < -\frac{8}{5}$

(iii)

Class	Frequency (f)	x	f
0-20	15	10	150
20-40	20	30	600
40-60	30	50	1500
60-80	a	70	70a
80-100	10	90	900
	Σf = 75 + a		$\sum fx$ = 3150 + 70a

Given mean = 49

$\therefore \frac{\sum fx}{\sum f}$ = 49

or, $\frac{3150+70a}{75+a}$ = 49

3150 + 70a = 3675 + 49a

70a - 49a = 3675 - 3150

21a = 525

a = 25

5. Question 5

(i) We know that two matrices are said to be equal if each matrix has the same number of rows and same number of columns. Corresponding elements within each matrix are equal.

Given: $\begin{bmatrix} x+y & a-b \\ a+b & 2x-3y \end{bmatrix} = \begin{bmatrix} 5 & 3 \\ -1 & -5 \end{bmatrix}$

x + y = 5 ...(i),

2x - 3y = -5 ...(ii)

a - b = 3 ...(iii)

a + b = -1 ...(iv)

Solving eqn (i) and (ii)

$$\begin{array}{r} 2x + 2y = 10 \\ 2x - 3y = -5 \\ - \quad + \qquad + \\ \hline 5y = 15 \end{array}$$

$\Rightarrow$ y = 3

Putting the value of y in eqn (i)

$\therefore$ x + 3 = 5

$\Rightarrow$ x = 2

Again solving eqn (iii) and (iv)

$$\begin{array}{r} a - b = 3 \\ a + b = -1 \\ \hline 2a \quad = 2 \end{array}$$

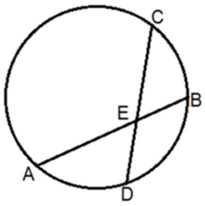
$\Rightarrow$ a = 1

Putting the value of a in eqn (iv)

$\Rightarrow$ 1 + b = -1

b = -2

$\Rightarrow$ x = 2, y = 3, a = 1, b = -2

(ii) 

Given, two chords AB and CD are intersect each other at point E.

AB = 9 cm

AE = 4 cm

ED = 6 cm

So, BE = AB - AE = 9 - 4 = 5

So, AE $\times$ EB = DE $\times$CE

$\Rightarrow$ 4 $\times$ 5 = 6 $\times$ CE

$\therefore$ CE = $\frac{4\times5}{6}$ = 3.34

(iii) g(x) = 0

x - 7 = 0, x = 7

By factor theorem, g(x) will be a factor of f(x)

if f(7) = 0

Now, f(7) = $(7)^3 - 6 \times (7)^2 - 19 \times 7 + 84$

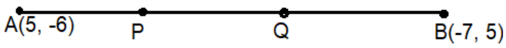
= 343 - 294 - 133 + 84

= 427 - 427 = 0

f(7) = 0, So, g(x) is a factor of f(x).

6. Question 6

(i) Let P and Q be the points of trisection of AB.



Given points be A(5, -6) and B(-7, 5)

P divides AB in the ratio 1 : 2

[By section formula, $\frac{mx_2+nx_1}{m+n}$, $\frac{my_2+ny_1}{m+n}$]

the coordinate P are

$\left(\frac{1\times(-7)+2\times5}{1+2}, \frac{1\times5+2\times(-6)}{1+2}\right)$ = $\left(\frac{-7+10}{3}, \frac{5-12}{3}\right)$ = $\left(1, \frac{-7}{3}\right)$

$P\left(1, \frac{-7}{3}\right)$

Q divides AB in the ratio 2 : 1 then the coordinates of Q are

$\left(\frac{2\times(-7)+4\times5}{2+1}, \frac{2\times5+1\times(-6)}{2+1}\right) = \left(\frac{-14+5}{3}, \frac{10-6}{3}\right) = \left(-3, \frac{4}{3}\right)$

$Q\left(-3, \frac{4}{3}\right)$

Hence, the points of trisection of AB are $P(1, -\frac{7}{3})$ and $Q\left(-3, \frac{4}{3}\right)$.

(ii) i. LHS = $\sin^4\theta + \cos^4\theta = 1 - 2\sin^2\theta\cos^2\theta$

$= (\sin^2\theta + \cos^2\theta)^2 - 2\cdot\sin^2\theta\cos^2\theta$ [$\because a^2 + b^2 = (a+b)^2 - 2ab$]

$= 1^2 - 2\sin^2\theta\cos^2\theta = 1 - 2\sin^2\theta\cos^2\theta$ [$\because \sin^2 A + \cos^2 A = 1$]

= RHS

Hence proved.

ii. $\frac{1}{\operatorname{cosec}\theta-\cot\theta} - \frac{1}{\sin\theta} = \frac{1}{\sin\theta} - \frac{1}{\operatorname{cosec}\theta+\cot\theta}$ is true

if $\frac{1}{\operatorname{cosec}\theta-\cot\theta} + \frac{1}{\operatorname{cosec}\theta+\cot\theta} = \frac{1}{\sin\theta} + \frac{1}{\sin\theta}$ is true

i.e. if $\frac{(\operatorname{cosec}\theta+\cot\theta)+(\operatorname{cosec}\theta-\cot\theta)}{(\operatorname{cosec}\theta-\cot\theta)(\operatorname{cosec}\theta+\cot\theta)} = \frac{2}{\sin\theta}$ is true

i.e. if $\frac{2\operatorname{cosec}\theta}{\operatorname{cosec}^2\theta-\cot^2\theta} = 2\operatorname{cosec}\theta$ is true

i.e. if $\frac{2\operatorname{cosec}\theta}{1} = 2\operatorname{cosec}\theta$ is true. [$\because \operatorname{cosec}^2\theta - \cot^2\theta = 1$]

which is true.

Hence proved.

(iii) Let total work be 1 and let total work completed in days.

work dream in 1 day = $\frac{\text{Total work}}{\text{Number of days to complete work}}$

$\frac{1}{n}$

This is the work done by 150 workers

work done by 1 worker in one day = $\frac{1}{150n}$

Number of workers	work done per worker in 1 day	Total work done in 1 day
150	$\frac{1}{150n}$	$\frac{150}{150n}$
146	$\frac{1}{150n}$	$\frac{146}{150n}$
142	$\frac{1}{150n}$	$\frac{142}{150n}$

Given that, In this manner, it took 8 more days to finish the work i.e. work finished in (n + 8) days.

$\therefore \frac{150}{150n} + \frac{146}{150n} + \frac{142}{150n}$ + ... + (n + 8) terms = 1

$\frac{1}{150n}$[150 + 146 + 142 + ... + (n + 8) terms] = 1

$\Rightarrow$ 150 + 146 + 142 + ... +(n + 8) terms = 150n

Now a = 150 d = 146 - 150

= -4

$\because$ diff. is equal $\therefore$ It forms A.P.

$\because S_n = \frac{n}{2}[2a + (n-1)d]$

$\therefore$ 150 + 146 + 142 +.. (n = 8) terms = 150n becomes.

$\frac{n+8}{2}$[2(150) + (n + 8 - 1)] = 150n

$\Rightarrow \frac{(n+8)}{2} \times 2$ [150 - 2(n + 7)] = 150n

$\Rightarrow$ (n + 8)(150 - 2n - 14) = 150n

$\Rightarrow$ (n + 8)(136 - 2n) = 150n

$\Rightarrow 136n - 2n^2 + 1088 - 16n = 150n$

$\Rightarrow 2n^2 - 120n - 1088 + 150n = 0$

$\Rightarrow 2n^2 + 30n - 1088 = 0$

$\Rightarrow n^2 + 15n - 544 = 0$

$\Rightarrow n^2 + 32n - 17n - 544 = 0$

$\Rightarrow$ n(n + 32) - 17(n + 32) = 0

$\Rightarrow$ (n + 32)(n - 17) = 0

$\Rightarrow$ n + 32 = 0 or n = 17

n = -32 n = 17

Reject n = -32 as n should be natural no.

n = 17 work was complete in 17 + 8 = 25 days.

7. Question 7

(i) Let the two digit no. be 10x + y

product of their digits

i.e., xy = 6

y = $\frac{6}{x}$ ...(i)

According to the question

10x + y + 9 = 10y + x

9x - 9y + 9 = 0

x - y = -1 ...(ii)

Substituting the value of y from equal (i),

$\frac{x}{1} - \frac{6}{x}$ = -1

$\frac{x^2-6}{x}$ = -1

x^2 - 6 = -x

x^2 + x - 6 = 0

x^2 + 3x - 2x - 6 = 0

x(x + 3) - 2(x + 3) = 0

(x - 2)(x + 3) = 0

x = 2, x = - 3 (according to question rejected as digits are never negative)

put the value of x in eqn. (i)

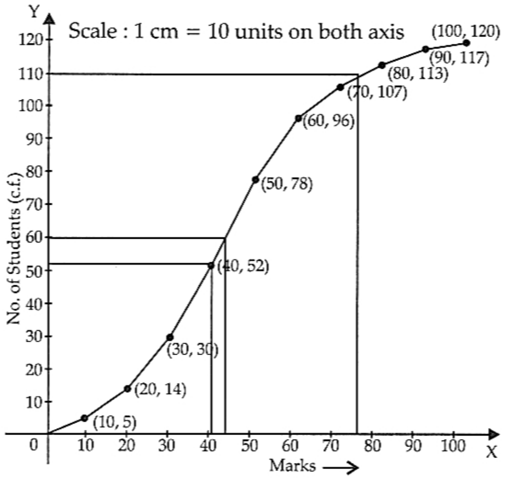
y = $\frac{6}{x} = \frac{6}{2}$ = 3

Thus, x = 2 and y = 3

Hence, the required no. = 10 $\times$ 2 + 3 = 23

(ii)

C.I.	**f**	**c.f**
0 - 10	5	5
10 - 20	9	14
20 - 30	16	30
30 - 40	22	52
40 - 50	26	78
50 - 60	18	96
60 - 70	11	107
70 - 80	6	113
80 - 90	4	117
90 - 100	3	120
	N = 120	



i. Given, n = 120 which is even
   $\therefore$ Median = $\frac{120}{2}$ th term
   = 60th term
   Median = 43
ii. The number of students who obtained more than 75% marks in test = 120 - 110 = 10.

8. Question 8

(i) Let E and F denote the events that Niharika and Shreya win the match, respectively. It is clear that, if Niharika wins the match, then Shreya losses the match and if Shreya wins the match, then Niharika losses the match. Thus, E and F are complementary events.

$\therefore$ P(E) + P(F) = 1

Since, probability of Niharika 's winning the match, i.e. P(E) = 0.62

$\therefore$ Probability of Shreya's winning the match,

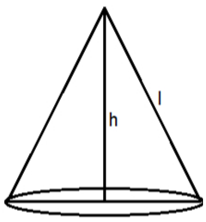
P(F) = P (Niharika losses the match)

= 1 - P(E) $[\because P(E) + P(F) = 1]$

= 1 - 0.62 = 038

(ii) Given:

h = 5m, d = 24 m, r = 12 m



$\therefore l = \sqrt{h^2 + r^2}$

$= \sqrt{5^2 + 12^2}$

l = 13 m

Convas Required = C.S.A of conical tent

$= \pi r l$

$= \frac{22}{7} \times 12 \times 13$

Convas Required = 490.28 m^2

Total cost = 490.28 $\times$ 14

= ₹6814

Hence, total cost of convas used = ₹6814

(iii) i. $\angle ABC + \angle ADC = 180^\circ$ (Opposite angles of cyclic quadrilateral)
   $93^\circ + \angle ADC = 180^\circ$
   $\angle ADC = 180 - 93^\circ$
   $\angle ADC = 180 - 93^\circ$
   = 87^o
ii. $\angle CAD = \angle ECD$ (Alternate segment theorem)
   $\therefore \angle CAD = 35^\circ$

iii. In $\triangle ADC, \angle ACD + \angle CAD + \angle ADC$ = 180^o (Sum of internal angles of a triangle = 180°)

$\angle ACD + 35^\circ + 87^\circ = 180^\circ$

$\angle ACD = 180^\circ - (35^\circ + 87^\circ)$

$\angle ACD = 180^\circ - 122^\circ$

$\angle ACD = 58^\circ$

9. Question 9

(i) i. A = {x : 3 < 2x - 1 < 9, x $\in$ R}

3 < 2x - 1 < 9

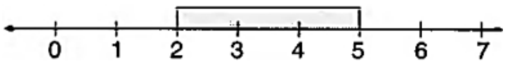
3 + 1 < 2x - 1 + 1 < 9 + 1

4 < 2x < 10

$\frac{4}{2} < \frac{2x}{2} < \frac{10}{2}$

2 < x < 5

A = (2, 5) $\in$ R



B = {x : 11 $\leq$ 3x + 2 $\leq$ 23, x $\in$ R}

11 $\leq$ 3x + 2 $\leq$ 23

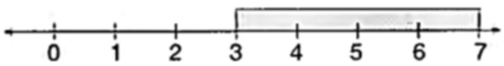
11 - 2 $\leq$ 3x + 2 - 2 $\leq$ 23 - 2

9 $\leq$ 3x $\leq$ 21

$\frac{9}{3} \leq \frac{3x}{3} \leq \frac{21}{3}$

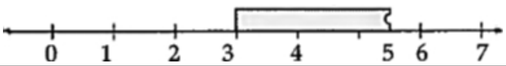
3 $\leq$ x $\leq$ 7

B = [3, 7] $\in$ R



ii. A $\cap$ B = (2, 5) $\cap$ [3, 7]

= [3, 5)



(ii)

Class Interval	**Required**	**mid value**	$\mathbf{f_i \times x_i}$
11 - 13	3	12	36
13 - 15	6	14	84
15 - 17	9	16	144
17 - 19	13	18	234
19 - 21	f	20	20f
21 - 23	5	22	110
23 - 25	4	24	96
	$\sum f_i = 40 + f$		$\sum(f_i \times x_i)$ = 704 + 20f

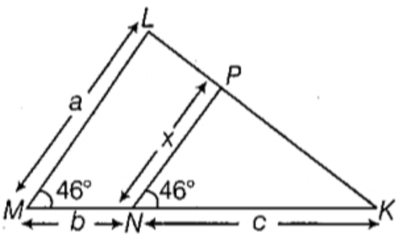
mean $(\bar{x}) = \frac{\sum(f_i \times x_i)}{\sum(f_i)}$

18 = $\frac{704+20f}{40+f}$

720 + 18f = 704 + 20f

f = 8

(iii)



Given: In the given figure.

$\angle$LMN = $\angle$PNK = 46^o

$\Rightarrow$ LM || PN (as corresponding angles are equal)

Now consider $\triangle$LMK and $\triangle$PNK

$\angle$LMK = $\angle$PNK (corresponding angles are equal)

$\angle$LKM = $\angle$PKN (common)

$\therefore \triangle$LMK $\sim \triangle$PNK (AA similarity)

$\frac{ML}{NP} = \frac{MK}{NK}$

$\frac{a}{x} = \frac{b+c}{c}$

x = $\frac{b+c}{c}$

Hence we get the result x = $\frac{ac}{b+c}$

10. Question 10

(i) Let the present age of A and B are 7x and 8x respectively.

Then, 6 years ago their ages are 7x - 6 and 8x - 6

So, $\frac{7x-6}{8x-6} = \frac{5}{6}$

$\Rightarrow$ 6(7x - 6) = 5(8x - 6)

$\Rightarrow$ 42x - 36 = 40x - 30

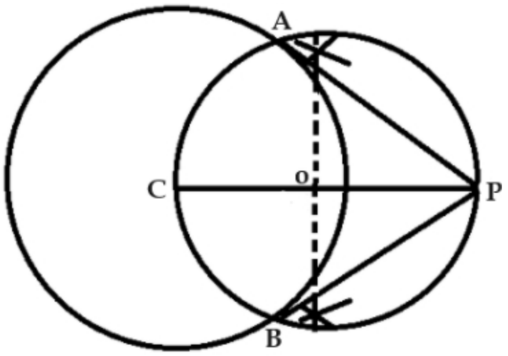
$\Rightarrow$ 42x - 40x = -30 + 36

2x = 6

$\therefore$ x = $\frac{6}{2}$ = 3

Hence, the present age of A and B are 21 and 24.

(ii)
i. Draw a circle with radius 6 cm and centre C.
ii. Take a point P at 10 cm from centre and join CP.
iii. Draw perpendicular bisector of CP which cuts CP at O.
iv. Take O as centre and OC as radius draw a circle which cuts the previous circle at A and B.
v. Join PA and PB.
vi. PA and PB are required tangents.



(iii) We have, $\angle$RPQ = 60^{o} and $\angle$PQT = 30^{o}

and QR = 50 m

Let PT = x m and PQ = y m

In $\triangle$PQR,

tan 60^{o} = $\frac{QR}{PQ}$

$\Rightarrow \sqrt{3} = \frac{50}{y}$

or y = $\frac{50}{\sqrt{3}}$ ...(i)

In $\triangle$PQT,

tan 30^{o} = $\frac{PT}{PQ}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{x}{y}$

or x = $\frac{y}{\sqrt{3}}$ ...(ii)

From eq. (i) and (ii), we get

x = $\frac{50}{\sqrt{3}\cdot\sqrt{3}} = \frac{50}{3}$ = 16.66

= 17 m (correct to the nearest meter)

ICSE 2025 EXAMINATION

Sample Question Paper - 2

Mathematics

Time Allowed: 2 hours and 30 minutes **Maximum Marks: 80**

General Instructions:

- Answers to this Paper must be written on the paper provided separately.
- You will not be allowed to write during the first 15 minutes.
- This time is to be spent reading the question paper.
- The time given at the head of this Paper is the time allowed for writing the answers.
- **Attempt all questions from Section A and any four questions from Section B**.
- All work, including rough work, must be clearly shown and must be done on the same sheet as the rest of the answers.
- Omission of essential work will result in a loss of marks.
- The intended marks for questions or parts of questions are given in brackets [ ]
- Mathematical tables are provided.

Section A

1. **Question 1 Choose the correct answers to the questions from the given options:** **[15]**

(a) If the cost of an article is ₹ 25,000 and CGST paid by the owner is ₹ 2250, the rate of GST is **[1]**

a) 18% b) 15%

c) 9% d) 10%

(b) From a group of Saras birds, one-fourth of the number are moving about in lotus plants, one-ninth coupled with one-fourth as well as 7 times the square root of the total number are moving on a hill, while 56 birds are sitting in the Bakula trees. Then, what is the total number of birds? **[1]**

a) 629 b) 675

c) 576 d) 567

(c) If x + 1 is a factor of $3x^3 + kx^2 + 7x + 4$, then the value of k is **[1]**

a) 14 b) 0

c) 6 d) -6

(d) If $A = \begin{bmatrix} 5 & 5 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 \\ 5 & 5 \end{bmatrix}$ and $A^n = \begin{bmatrix} 5^{200} & 5^{200} \\ 0 & 0 \end{bmatrix}$, then the value of n is **[1]**

a) 100 b) 75

c) 25 d) 50

(e) If a, b and c are respectively the pth, qth and rth terms of a GP, then the value of **[1]**

(q - r) log a + (r - p) log P + (p - g) log c is

a) log abc
b) log bc
c) log ab
d) 0

(f) A point M is reflected in X-axis to M'(4, -5). M'' is the image of M, when reflected in the Y-axis. The coordinates of M''' when M'' is reflected in the origin, is **[1]**

a) (-4, -5)
b) (-4, 5)
c) (4, 5)
d) (4, -5)

(g) Diagonal AC of a rectangle ABCD is produced to the point E such that AC : CE = 2 : 1, AB = 8 cm and BC = 6 m. The length of DE is **[1]**

a) $3\sqrt{17}$ cm
b) 15 cm
c) 13 cm
d) $2\sqrt{19}$ cm

(h) A hollow cone of radius 6 cm and height 8 cm is vertical standing at the origin, such that the vertex of the cone is at the origin. Some pipes are hanging around the circular base of the cone, such that they touch the surface of the graph paper. Then, the total surface area of the formed by the figure will be **[1]**

a) 494.68 cm^2
b) 484.98 cm^2
c) 489.84 cm^2
d) 948.84 cm^2

(i) Find the range of values of x which satisfy the inequation, $(x + 1)^2 - (x - 1)^2 < 6$. **[1]**

a) $\left(-\infty, \frac{3}{2}\right)$
b) $\left(\frac{3}{2}, \infty\right)$
c) $\left(-\infty, -\frac{3}{2}\right)$
d) $\left(-\infty, -\frac{3}{2}\right) \cup \left(\frac{3}{2}, \infty\right)$

(j) The probability that the minute hand lies from 5 to 15 min in the wall clock, is **[1]**

a) $\frac{1}{6}$
b) $\frac{5}{6}$
c) $\frac{1}{5}$
d) $\frac{1}{10}$

(k) If $\begin{bmatrix} a^x \\ a^{-x} \end{bmatrix} \begin{bmatrix} 1 & 2 \end{bmatrix} = \begin{bmatrix} p & a^{-2} \\ q & \log_2 2 \end{bmatrix}$, a > 0, then a^{p-q} is equal to **[1]**

a) $4^{\frac{3}{2}}$
b) 1
c) $2^{\frac{-3}{2}}$
d) $2^{\frac{3}{2}}$

(l) Suppose PQ be a pole, whose coordinates are P(1, 3) and 0(3, 3) and A be the position of a man whose coordinates are (1, 1). **[1]**

i. If a pole makes an angle of elevation to the point A, then the angle θ is
ii. Also, if we shift the origin at (1, 1), then the angle θ is

a) 75, 45°
b) 45°, 60°
c) 45°, 90
d) 45°, 45°

(m) If P, Q, S and R are points on the circumference of a circle of radius r, such that PQR is an equilateral triangle and PS is a diameter of the circle. Then, the perimeter of the quadrilateral PQSR will be **[1]**

a) $2(\sqrt{3} + 1)$r
b) $2\sqrt{3} + r$

c) 2r | d) $2\sqrt{3}r$

(n) If the ratio of mode and median of a certain data is 6 : 5, then the ratio of its mean and median is **[1]**

a) 10 : 9 | b) 9 : 10

c) 10 : 8 | d) 8 : 10

(o) **Assertion (A):** a_n - a_{n-1} is not independent of n then the given sequence is an AP. **[1]**

Reason (R): Common difference d = a_n - a_{n-1} is constant or independent of n.

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

2. **Question 2** **[12]**

(a) Mr. Gupta opened a recurring deposit account in a bank. He deposited ₹ 2,500 per month for 2 years. At the time of maturity, he got ₹67,500. Find: **[4]**

i. the total interest earned by Mr. Gupta

ii. the rate of interest per annum.

(b) Find the third proportional to **[4]**

i. 16 and 36

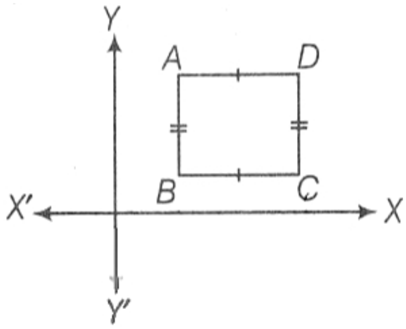
ii. $(x^2 + y^2 + xy)^2$ and $(x^3 - y^3)$

(c) Prove that: $\frac{(1+\cot\theta+\tan\theta)(\sin\theta-\cos\theta)}{\sec^3\theta-\operatorname{cosec}^3\theta} = \sin^2\theta\cos^2\theta$ **[4]**

3. **Question 3** **[13]**

(a) The internal and external diameters of a hollow hemispherical vessel are 7cm and 14 cm, respectively. The cost of silver plating of 1 sq cm surface is ₹ 0.60. Find the total cost of silver plating the vessel all over. **[4]**

(b) The side AB of a square ABCD is parallel to the Y-axis as shown in the given figure. **[4]**



Calculate

i. the slope of AD.

ii. the slope of BD.

iii. the slope of AC. [Given, $\tan(90^o + 0) = -\cot\theta$]

(c) Use graph paper to answer this question: **[5]**

i. The point P(2, - 4) is reflected about the line x = 0 to get the image Q. Find the coordinates of Q.

ii. Point Q is reflected about the line y = 0 to get the image R. Find the coordinates of R.

iii. Name the figure PQR.

iv. Find the area of figure PQR.

Section B

Attempt any 4 questions

4. **Question 4** **[10]**

(a) A shopkeeper bought an article with market price ₹1200 from the wholesaler at a discount of 10%. The shopkeeper sells this article to the customer on the market price printed on it. If the rate of GST is 6%, then find: **[3]**

i. GST paid by the wholesaler.

ii. Amount paid by the customer to buy the item.

(b) The sum of the squares of two consecutive odd positive integers is 290. Find them. **[3]**

(c) Draw a Histogram for the given data, using a graph paper: **[4]**

Weekly Wages (in ₹)	**No. of People**
3000-4000	4
4000-5000	9
5000-6000	18
6000-7000	6
7000-8000	7
8000-9000	2
9000-10000	4

Estimate the mode from the graph.

5. **Question 5** **[10]**

(a) Evaluate, $\begin{bmatrix} 4\sin 30^\circ & 2\cos 60^\circ \\ \sin 90^\circ & 2\cos 0^\circ \end{bmatrix}\begin{bmatrix} 4 & 5 \\ 5 & 4 \end{bmatrix}$. **[3]**

(b) O is the circumcentre of the $\triangle$ABC and D is mid-point of the base BC. Prove that $\angle$BOD = $\angle$A. **[3]**

(c) Use factor theorem to factorise $6x^3 + 17x^2 + 4x - 12$ completely. **[4]**

6. **Question 6** **[10]**

(a) Calculate the ratio in which the line joining A (-4, 2) and B(3, 6) is divided by P(x, 3). Also, find **[3]**

i. x

ii. length of AP

(b) Prove that: $1 - \frac{\cos^2\theta}{1+\sin\theta} = \sin\theta$ **[3]**

(c) Sum of the first n terms of an AP is $5n^2 - 3n$. Find the AP and also find its 16th term. **[4]**

7. **Question 7** **[10]**

(a) A grassy land is in the shape of a right triangle. The hypotenuse of the land is 1 m more than twice the shortest side. If the third side is 7 m more than the shortest side, find the sides of the grassy land. **[5]**

(b) The marks obtained by 100 students in a Mathematics test are given below **[5]**

Marks	**Number of students**
0-10	3
10-20	7
20-30	12

30-40	17
40-50	23
50-60	14
60-70	9
70-80	6
80-90	5
90-100	4

Draw an ogive for the given distribution on a graph sheet, (use a scale of 2 cm = 10 units on both axes). Use the ogive to estimate the

i. median.

ii. lower quartile.

iii. number of students who obtained more than 85% marks in the test.

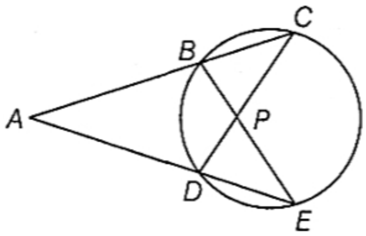
iv. number of students who did not pass in the test, if the pass percentage was 35.

8. **Question 8** **[10]**

(a) A number is selected at random from first 50 natural numbers. Find the probability that it is a multiple of 3 and 4. **[3]**

(b) How many solid spheres of diameter 6 cm are required to be melted to form a cylindrical solid of height 45 cm and diameter 4 cm? **[3]**

(c) In the given figure, AC = AE. **[4]**



Show that

i. CP = EP

ii. BP = DP

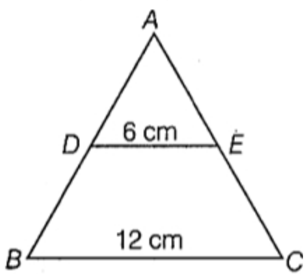
9. **Question 9** **[10]**

(a) Solve the following inequation and represent the solution set on the number line. **[3]**

$\frac{3x}{5} + 2 < x + 4 \leq \frac{x}{2} + 5, x \in R$

(b) Mode and mean of a data are 12k and 15k respectively. Find the median of the data. **[3]**

(c) In the given figure, if DE || BC, find the ratio of ar ($\triangle$ADE) and ar (DECB). **[4]**



10. **Question 10** **[10]**

(a) Find the fourth proportional to $(a^3 + 8)$, $(a^4 - 2a^3 + 4a^2)$ and $(a^2 - 4)$. **[3]**

(b) Use a ruler and a pair of compasses to construct a $\triangle ABC$, in which BC = 4.2 cm, $\angle ABC = 60^o$ and AB = 5 cm. Construct a circle of radius 2 cm to touch both the arms of $\angle ABC$ of $\angle ABC$. **[3]**

(c) A man observes the angle of elevation of the top of a building to 30^o. He walks towards it in a horizontal line though its base. On covering 60 m, the angle of elevation changes to 60^o. Find the height of the building correct to the nearest metre. **[4]**

Solution

Section A

1. Question 1 Choose the correct answers to the questions from the given options:

(i) **(a)** 18%

Explanation: {

C.P. = ₹ 25,000, CGST = ₹ 2250

$\therefore$ GST = 2 $\times$ ₹ 2250 = ₹ 4500

let rate of GST = r%

$\therefore$ r% of ₹ 25,000 = ₹ 4500

$\Rightarrow$ r = (4500 $\times$ 100) $\div$ 25000 = 18%

(ii) **(c)** 576

Explanation: {

Let the total number of Saras birds be x.

Then, number of Saras birds moving in lotus plants = $\frac{x}{4}$

Number of Saras birds moving on a hill = $\frac{x}{9}$ + $\frac{x}{4}$ + 7 $\sqrt{x}$

Number of Saras birds sitting on the Bakula trees = 56

According to the question,

$\frac{x}{4}+\frac{x}{9}+\frac{x}{4}+7\sqrt{x}+56=x$

$\Rightarrow 7\sqrt{x} = x - \frac{x}{4} - \frac{x}{9} - \frac{x}{4} - 56$

$\Rightarrow 7\sqrt{x} = \frac{36x-9x-4x-9x}{36} = 56$

$\Rightarrow 7\sqrt{x}=\frac{7x}{18} - 56 \Rightarrow \sqrt{x}=\frac{x}{18} - 8$

$\Rightarrow x = \frac{x^2}{324} + 64 - \frac{8x}{9}$ [squaring on both sides]

$\Rightarrow x = \frac{x^2+20736-288x}{324}$

$\Rightarrow$ 324x = x^2 + 20736 - 288x

$\Rightarrow$ x^2 - 612x + 20736 = 0

$\Rightarrow$ x^2 - 36x - 576x + 20736 = 0 [splitting the middle term]

$\Rightarrow$ x(x - 36) - 576(x - 36) = 0

$\Rightarrow$ (x - 36)(x - 576) = 0

$\Rightarrow$ x - 36 = 0 or x - 576 = 0

$\Rightarrow$ x = 576 or x = 36

Here, x = 36 is not possible, because if there are only 36 birds, then 56 cannot be on the trees.

Thus, total number of Saras birds is 576.

(iii) **(c)** 6

Explanation: {

Let f(x) = $3x^3 + kx^2 + 7x + 4$

As x + 1 is a factor of f(x), f(-1) = 0

$\Rightarrow 3(-1)^3 + k(-1)^2 + 7(-1) + 4 = 0$

$\Rightarrow$ -3 + k - 7 + 4 = 0

$\Rightarrow$ k = 6

(iv) **(a)** 100

Explanation: {

We have, $A=\begin{bmatrix}5&5\\0&0\end{bmatrix}\begin{bmatrix}0&0\\5&5\end{bmatrix}=\begin{bmatrix}25&25\\0&0\end{bmatrix}=\begin{bmatrix}5^2&5^2\\0&0\end{bmatrix}$

$\therefore A^2 = A \cdot A = \begin{bmatrix}5^2&5^2\\0&0\end{bmatrix}\begin{bmatrix}5^2&5^2\\0&0\end{bmatrix}$

$=\begin{bmatrix}5^4&5^4\\0&0\end{bmatrix}$

$A^n = \begin{bmatrix} 5^{2n} & 5^{2n} \\ 0 & 0 \end{bmatrix}$

Thus, $\begin{bmatrix} 5^{2n} & 5^{2n} \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 5^{200} & 5^{200} \\ 0 & 0 \end{bmatrix}$

$\Rightarrow 5^{2n} = 5^{200}$

$\Rightarrow$ 2n = 200

$\Rightarrow$ n = 100

(v) **(d)** 0

Explanation: {

Let A be the first term and R be the common ratio of the given GP.

Then, a = pth term $\Rightarrow$ a = $AR^{P - 1}$

$\Rightarrow$ log a = log A + (p - 1)log R ...(i)

b = qth term $\Rightarrow$ b = AR^{q-1}

$\Rightarrow$ log b = log A + (q - 1) log R ...(ii)

c = rth term $\Rightarrow$ c = AR^{r-1}

$\Rightarrow$ log c = log A + (r - 1) log R ...(iii)

$\Rightarrow$ Now, consider {q - r) log a + (r - p)log b + (p - q)log c

=(q - r){log A + (p - 1)log R} + (r - p){log A + (q - 1)log R} [from Eqs. (i), (ii) and (iii)]

= log A{(q - r)+ (r - p) + (p - q)} + log R{(p - 1) (q - r) + (q - 1)(r - p) + (r - 1)(p - q)}

= (log A) 0 + {p(q - r) + q (r - p)+ r(p - q) - (q - r) - (r - p) - (p - q)} log R|

= (log A)0 + (log R) 0 = 0

(vi) **(d)** (4, -5)

Explanation: {

Since, the image of any point (x, y) under X-axis is (x, - y).

$\therefore$ Coordinate of M $\equiv$ (4, 5)

Since, the image of any point (x, y) under Y-axis is (-x, y).

$\therefore$ Coordinate of M" $\equiv$ (4, -5)

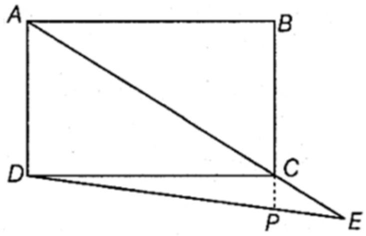
Since, the image of any point (x, y) under origin is (-x, -y).

$\therefore$ Coordinate of M''' = (4, - 5)

(vii) **(a)** $3\sqrt{17}$ cm

Explanation: {

Given AB = 8 cm and BC = 6 cm



$\therefore$ AC = $\sqrt{8^2 + 6^2}$ = 10 cm

Also, given AC : CE = 2 : 1

Now, produce BC to meet DE at the point P as CP is parallel to AD,

$\triangle ECP \sim \triangle EAD$ ...(i)

$\Rightarrow \frac{CP}{AD} = \frac{CE}{AE} \Rightarrow \frac{CP}{6} = \frac{1}{3}$ ...(ii)

$\Rightarrow$ CP = 2 cm

Also, $\triangle$CPD is right triangle.

$\therefore$ DP = $\sqrt{CD^2 + CP^2}$

$= \sqrt{68} = 2\sqrt{17}$ cm

But DP = PE = 2 : 1 [from Eq.(i)]

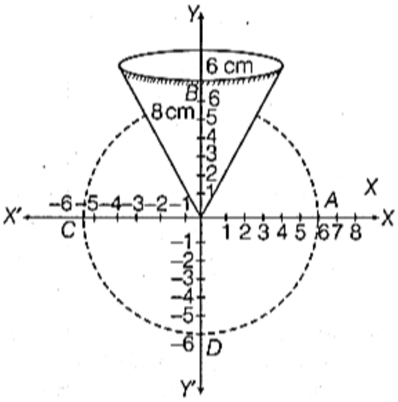
$\therefore$ PE = $\sqrt{17}$ cm

Thus, DE = DP + PE = $2\sqrt{17} + \sqrt{17} = 3\sqrt{17}$ cm

(viii) **(c)** 489.84 cm^2

Explanation: {

According to the given information, a shape of figure is shown below



When the hanging pipes touches the surface paper, a circular shape ABCD is formed on the graph paper. The size of circle ABCD is equal to the size of circular base of the cone.

$\therefore$ Radius of the circle ABCD is 6 cm.

Hence, the coordinates of A, B, C and Dare (6, 0), (0, 6), (-6, 0) and (0, -6), respectively.

The figure formed in the given information is cylindrical in outer surface and conical in the inner surface. Now, total surface area of the figure

= Curved surface area of the cylinder + Curved surface area of the cone

= $2\pi rh + \pi rl = \pi r(2h + l)$

= $\pi r(2h + \sqrt{r^2 + h^2})$

= $3.14 \times 6\left(2 \times 8 + \sqrt{6^2 + 8^2}\right)$

= $18.84(16 + \sqrt{36 + 64})$

= $18.84(16 + \sqrt{100}) = 18.84(16 + 10)$

= $18.84 \times 26 = 489.84$ cm^2

(ix) **(a)** $\left(-\infty, \frac{3}{2}\right)$

Explanation: {

We have, $(x + 1)^2 - (x - 1)^2 < 6$

$\Rightarrow (x^2 + 1 + 2x) - (x^2 + 1 - 2x) < 6$ [$\because (a \pm b)^2 = a^2 + b^2 \pm 2ab$]

$\Rightarrow 4x < 6$

$\Rightarrow x < \frac{6}{4}$

$\Rightarrow x < \frac{3}{2}$

$\Rightarrow x \in \left(-\infty, \frac{3}{2}\right)$

(x) **(a)** $\frac{1}{6}$

Explanation: {

In a wall clock, the minute hand cover the 60 min in on complete round.

$\therefore$ Total number of possible outcomes = 60

The minute hand cover the time from 5 to 15 min,

Number of outcomes favourable to E

= Distance from 5 to 15 min = 10

$\therefore$ Required probability = $\frac{10}{60} = \frac{1}{6}$

(xi) **(c)** $2^{\frac{-3}{2}}$

Explanation: {

We have, $\begin{bmatrix} a^x \\ a^{-x} \end{bmatrix} \begin{bmatrix} 1 & 2 \end{bmatrix} = \begin{bmatrix} p & a^{-2} \\ q & \log_2 2 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} a^x & 2a^x \\ a^{-x} & 2a^{-x} \end{bmatrix} = \begin{bmatrix} p & a^{-2} \\ q & 1 \end{bmatrix}$ $\left[\because \log_2 2 = \frac{\log 2}{\log 2} = 1\right]$

On comparing the corresponding elements both sides, we get

$\Rightarrow$ $a^x = p$ ...(i)

$\Rightarrow$ $2a^x = a^{-2}$ ...(ii)

$\Rightarrow$ $a^{-x} = q$ ...(iii)

and $2a^{-x} = 1$ ...(iv)

On multiplying Eqs. (ii) and (iv), we get

$4a^{x-x} = a^{-2}$

$\Rightarrow$ $4a^0 = a^{-2} \Rightarrow 4 = a^{-2} \Rightarrow 4 = \frac{1}{a^2}$

$\Rightarrow$ $a^2 = \frac{1}{4} \Rightarrow a = \frac{1}{2}$ [$\because$ a > 0]

Now, $a^{p-q} = a^{a^x - a^{-x}}$ [from Eqs. (i) and (iii)]

$= a^{\frac{1}{2}a^{-2} - \frac{1}{2}} = a^{2-\frac{1}{2}}$ [from Eqs. (ii) and (iv)]

$= a^{\frac{1}{2}\cdot 4 - \frac{1}{2}}$ [$\because$ $a^{-2} = 4$]

$= \left(\frac{1}{2}\right)^{\frac{3}{2}} = 2^{\frac{-3}{2}}$

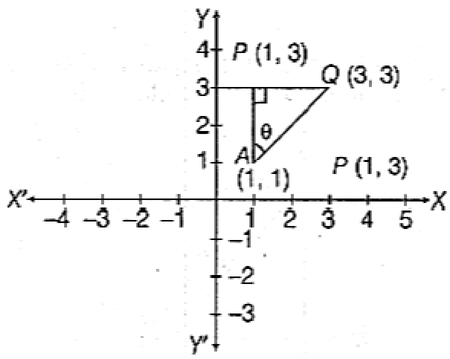
(xii) **(d)** 45°, 45°

Explanation: {

Given, coordinates of pole be P(1, 3) and Q (3, 3) and A(1, 1) be the position of man

i. Now, AP = $\sqrt{(1-1)^2 + (3-1)^2}$ [$\because$ distance = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$]

= $\sqrt{0^2 + 2^2}$ = 2 units



and PO = $\sqrt{(3-1)^2 + (3-3)^2} = \sqrt{2^2 + 0^2}$ = 2 units

Now, in $\triangle$APQ, we have

tan θ = $\frac{PQ}{AP} \Rightarrow$ tan θ = $\frac{2}{2}$ = 1

$\Rightarrow$ θ = 45° [$\because$ tan 45° = 1]

ii. When we shift the origin at (1, 1), then the angle will remain same, i.e. θ = 45°.

(xiii) **(a)** $2(\sqrt{3} + 1)r$

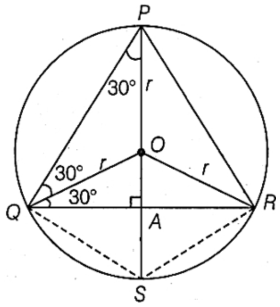
Explanation: {

As PQR is an equilateral triangle, hence PS will be perpendicular to QP and will divide it into 2 equal parts.

Since, $\angle$P and $\angle$S will be supplementary, so

$\angle$S = 120° and $\angle$QSA = $\angle$RSA = 60°

Now, PA = PQ cos 30° and OA = OQ sin 30° = $\frac{r}{2}$



$\Rightarrow$ AS = OA = $\frac{r}{2}$ and PA = PO + OA = r + $\frac{r}{2}$

Hence, PQ = $\frac{PA}{\cos 30^\circ} = \frac{r+\frac{r}{2}}{\frac{\sqrt{3}}{2}} = \sqrt{3}r$

In $\triangle$QAS, AS = QS cos $60^o \Rightarrow$ QS = $\frac{\frac{r}{2}}{\frac{1}{2}}$ = r

Since, AQ = AR, AS is common and $\angle$QAS = $\angle$RAS = 90^o

So, QS = RS.

$\therefore$ Perimeter of PQSP = 2(PQ + QS) = 2($\sqrt{3}$ + 1)r

(xiv) **(b)** 9 : 10

Explanation: {

We know that,

Mode = 3 Median - 2 Mean

On dividing both sides by median, we get

$\frac{\text{Mode}}{\text{Median}} = 3 - 2\frac{\text{Mean}}{\text{Median}}$

$\Rightarrow \frac{6}{5}$ = 3 - 2 $\frac{\text{Mean}}{\text{Median}}$ [$\because \frac{\text{mode}}{\text{median}} = \frac{6}{5}$, given]

$\Rightarrow \frac{6}{5}$ - 3 = -2 $\frac{\text{Mean}}{\text{Median}}$

$\Rightarrow \frac{6-15}{5}$ = -2 $\frac{\text{Mean}}{\text{Median}}$

$\Rightarrow \frac{-9}{5}$ = -2 $\frac{\text{Mean}}{\text{Median}}$

$\Rightarrow \frac{\text{Mean}}{\text{Median}} = \frac{9}{10}$

(xv) **(d)** A is false but R is true.

Explanation: {

We have, common difference of an AP

d = a_n - a_{n-1} is independent of n or constant.

So, A is false but R is true.

2. Question 2

(i) i. P = ₹ 2,500

n = 24 months

r = ?

M.V. = ₹ 67,500

Total money deposited in 2 years

= P $\times$ n

= 2,500 $\times$ 24

= 60,000

Total interest earned by Mr. Gupta = Maturity value - Money deposited

= 67,500 - 60,000

= ₹ 7,500

ii. M.V. = P $\times$ n + $\frac{P\times n(n+1)\times r}{2400}$

67,500 = 2500 $\times$ 24 + $\frac{2500\times 24\times 25\times r}{2400}$

67,500 = 60,000 + 625r

67,500 - 60,000 = 625r

7,500 = 625r

$\frac{7500}{625}$ = r

r = 12% p.a.

(ii) i. 16 and 36

Let the thid proportional to 16 and 36 be x.

$\Rightarrow$ 16, 36 and x ins continous proportion.

$\Rightarrow$ 16 : 36 = 36 : x

$\Rightarrow$ 16 $\times$ x = 36 $\times$ 36

$\Rightarrow x = \frac{36 \times 36}{16}$

$\Rightarrow$ x = 81

ii. $(x^2 + y^2 + xy)^2$ and $x^3 - y^3$

Let third proportional to $(x^2 + y^2 + xy)^2$ and $x^3 - y^3$ be x.

$\Rightarrow (x^2 + y^2 + xy)^2$, $x^3 - y^3$ and x are in continous proportion.

$\Rightarrow (x^2 + y^2 + xy)^2 : x^3 - y^3 = x^3 - y^3 : x$

$x = \frac{(x^3 - y^3)^2}{(x^2+y^2+xy)^2}$

$x = \frac{(x-y)^2 (x^2+y+xy)^2}{(x^2+y^2+xy)^2}$ $[\because x^3 - y^3 = (x - y)(x^2 + y^2 + xy)]$

$x = (x - y)^2$

(iii) L.H.S. $= \frac{(1+\cot\theta+\tan\theta)(\sin\theta-\cos\theta)}{\sec^3\theta-\operatorname{cosec}^3\theta}$

$= \frac{\left(1+\frac{\cos\theta}{\sin\theta}+\frac{\sin\theta}{\cos\theta}\right)(\sin\theta-\cos\theta)}{\frac{1}{\cos^3\theta}-\frac{1}{\sin^3\theta}}$

$= \frac{\left(\frac{1+\cos^2\theta+\sin^2\theta}{\sin\theta\cos\theta}\right)(\sin\theta-\cos\theta)}{\frac{\sin^3\theta-\cos^3\theta}{\sin^3\theta\cos^3\theta}}$ $\{\sin^2\theta + \cos^2\theta = 1\}$

$= \frac{(\sin\theta\cos\theta+1)(\sin\theta-\cos\theta)(\sin^3\theta\cos^3\theta)}{(\sin^3\theta-\cos^3\theta)\sin\theta\cos\theta}$

Since, we know,

$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

$= \frac{(\sin\theta\cos\theta+1)(\sin\theta-\cos\theta)(\sin^3\theta\cos^3\theta)}{(\sin\theta-\cos\theta)(\sin^2\theta+\cos^2\theta+\sin\theta\cos\theta(sin\theta\ cos\theta))}$

$= \frac{(\sin\theta\cos\theta+1)(\sin^2\theta\cos^2\theta)}{(1+\sin\theta\cos\theta)}$ $\{\because \sin^2\theta + \cos^2\theta = 1\}$

$= \sin^2\theta\cos^2\theta$ = RHS proved

3. Question 3

(i) Given, internal diameter of hollow hemispherical vessel = 7 cm

external diameter of a hollow hemispherical vessel = 14 cm

$r_1 = \frac{7}{2}$ cm

$r_2 = \frac{14}{2}$ = 7cm

Area of Ring = $\pi r_2^2 - \pi r_1^2$

Total area to be painted

$= 2\pi r_2^2 + 2\pi r_1^2 + \left(\pi r_2^2 - \pi r_1^2\right)$

$= 3\pi r_2^2 + \pi r_1^2$

$= \pi\left(3\pi r_2^2 + r_1^2\right)$

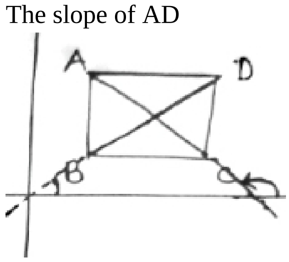
$= \pi\left[3 \times 49 + (3.5)^2\right]$ = 3.14 × (147 + 12.25)

= 500.045 cm^2

Hence, the total cost of silver painting the vessel = 500.045 × 0.6

= ₹ 300.027

(ii) i. The slope of AD



We know that the slope of any line parallel to x-axis is 0.

$\therefore$ The slope of AD = 0

ii. The slope of BD.

As ABCD in a square

$\therefore$ the diagonal BD makes an angle of 45^o with +ve direction of x-axis

$\therefore$ Slope of BD = tan 45^o = 1

iii. The diagonal AC make augle of 135^o with positive direction of x-axis.

$\therefore$ Slope of AC = tan 135^o

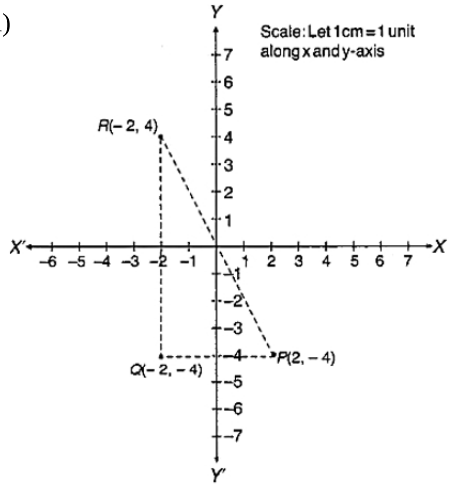
= tan (90 $\times$ 2 - 45)

= tan (-45)

= -tan 45

= -1

(iii)



i. The coordinates of Q are (- 2, - 4)

ii. The coordinates of R are (- 2, 4)

iii. PQR is Right angle Triangle

iv. Area of $\triangle$PQR = $\frac{1}{2} \times$ Base $\times$ height

$= \frac{1}{2} \times 4 \times 8$

= 2×8

= 16 sq. unit

Section B

4. Question 4

(i) i. C.P for the shopkeeper

= 1200 $\times$ $\frac{90}{100}$ = ₹1080

GST paid by the wholesaler

= 1080 $\times$ $\frac{60}{100}$ = ₹64.80

ii. S.P of the article = ₹1200

GST paid by the customer

= 1200 $\times$ $\frac{6}{100}$ = ₹72

Amount paid by the customer

= S.P. + GST = 1200 + 72 = ₹1272

(ii) Let the two consecutive odd no. be x and x + 2.

A/c question

$x^2 + (x + 2)^2 = 290$

$\Rightarrow x^2 + x^2 + 4x + 4 = 290$

$\Rightarrow 2x^2 + 4x - 286 = 0$

$\Rightarrow 2(x^2 + 2x - 143) = 0$

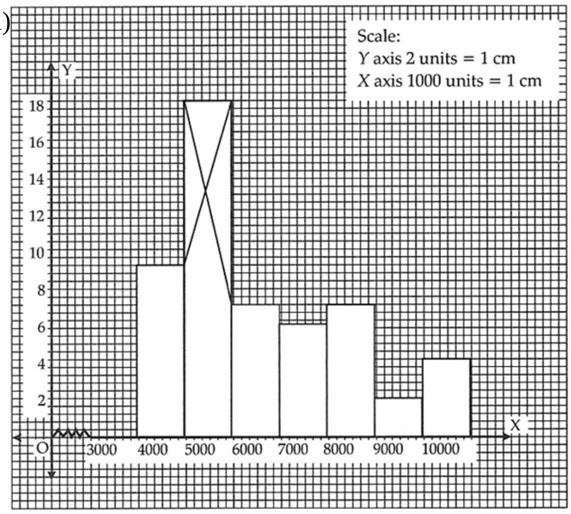
$\Rightarrow x^2 + 2x - 143 = 0$

$\Rightarrow x^2 + 13x - 11x - 143 = 0$

$\Rightarrow x(x + 13) - 11(x + 13) = 0$

x = 11 or x = -13 rejected

(iii)



In the given histogram, inside the highest rectangle, which represents the maximum frequency.

$\therefore$ Modal class = 5000 - 6000

Then, mode = 5500.

5. Question 5

(i) $\begin{bmatrix} 4\sin 30^\circ & 2\cos 60^\circ \\ \sin 90^\circ & 2\cos 0^\circ \end{bmatrix}\begin{bmatrix} 4 & 5 \\ 5 & 4 \end{bmatrix}$

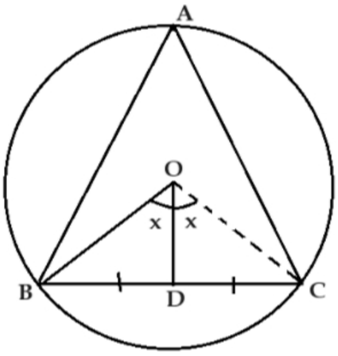
= $\begin{bmatrix} 4\times\frac{1}{2} & 2\times\frac{1}{2} \\ 1 & 2\times 1 \end{bmatrix}\begin{bmatrix} 4 & 5 \\ 5 & 4 \end{bmatrix}$[$\therefore$ sin 30° = cos 60° = $\frac{1}{2}$ and sin 90° = cos 0° = 1]

= $\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} 4 & 5 \\ 5 & 4 \end{bmatrix}$

= $\begin{bmatrix} 2\times 4+1\times 5 & 2\times 5+1\times 4 \\ 1\times 4+2\times 5 & 1\times 5+2\times 4 \end{bmatrix}$

= $\begin{bmatrix} 8+5 & 10+4 \\ 4+10 & 5+8 \end{bmatrix}$ = $\begin{bmatrix} 13 & 14 \\ 14 & 13 \end{bmatrix}$.

(ii)



To prove $\angle BOD = \angle A$

In $\triangle BDO$ and $\triangle CDO$

BD = DC (given)

OD = OD (common)

BO = OC (Radius)

By SSS

$\triangle BDO \cong \triangle CDO$

$\angle BOD = \angle COD$ = x

Now,

$\angle BAC = \frac{1}{2}\angle BOC$ (angle made by same arc)

$\angle BAC = \frac{1}{2}\times(2x)$

$\angle BAC$ = x

$\angle BAC$ = $\angle BOD$

$\angle A = \angle BOD$

Hence proved.

(iii) Let p(x) = $6x^3 - 17x^2 + 4x - 12$

Remainder p(- 2) = $6(-2)^3 + 17(-2)^2 + 4(-2) - 12$

= -48 + 68 - 8 - 12

= 68 - 68 = 0

$\therefore$ (x + 2) is a factor of given polynomial p(x)

$$
\begin{array}{l}
x+2 \,\big)\, 6x^3 + 17x^2 + 4x - 12 \,\big(\, 6x^2 + 5x - 6 \\
\quad\quad\;\; 6x^3 + 12x^2 \\
\quad\quad\;\; (-) \quad (-) \\
\hline
\quad\quad\quad\quad 5x^2 + 4x - 12 \\
\quad\quad\quad\quad 5x^2 + 10x \\
\quad\quad\quad\quad (-) \quad (-) \\
\hline
\quad\quad\quad\quad\quad - 6x - 12 \\
\quad\quad\quad\quad\quad - 6x - 12 \\
\quad\quad\quad\quad\quad\; (+) \;\; (+) \\
\hline
\quad\quad\quad\quad\quad\quad\quad 0
\end{array}
$$

$\therefore$ $6x^3 + 17x^2 + 4x - 12$

= $(x + 2)(6x^2 + 5x - 6)$

= $(x + 2)\{6x^2 + 9x - 4x - 6\}$

= (x + 2) {3x(2x + 3) - 2(2x + 3)}

= (x + 2)(2x + 3)(3x - 2)

6. Question 6

(i) i. By using section formula,

Let the ratio be k : 1

$y = \frac{m_2 y_1 + m_1 y_2}{m_1 + m_2}$

$3 = \frac{1 \times 2 + k \times 6}{k+1}$

3k + 3 = 2 + 6k

3k - 6k = 2 - 3

-3k = -1

k = $\frac{1}{3}$

Ratio = 1 : 3

$x = \frac{m_1 x_2 + m_2 x_1}{m_1 + m_2}$

$x = \frac{1 \times 3 + 3 \times -4}{1+3}$

$x = \frac{3-12}{4}$

$x = \frac{-9}{4}$

ii. Here coordinate of P is $\left(\frac{-9}{4}, 3\right)$

Length of, AP = $\sqrt{(y_2 - y_1)^2 + (x_2 - x_1)^2}$

$= \sqrt{(3-2)^2 + \left(-\frac{9}{4} - (-4)\right)^2}$

$= \sqrt{(1)^2 + \left(\frac{-9}{4} + 4\right)^2}$

$= \sqrt{(1)^2 + \left(\frac{-9+16}{4}\right)^2}$

$= \sqrt{(1)^2 + \left(\frac{7}{4}\right)^2}$

$= \sqrt{1 + \frac{49}{16}}$

$= \sqrt{\frac{65}{16}}$

$= \frac{\sqrt{65}}{4}$

(ii) LHS = $\frac{1}{1} - \frac{\cos^2\theta}{1+\sin\theta} = \frac{1+\sin\theta-\cos^2\theta}{1+\sin\theta}$

= $\frac{1+\sin\theta-\left(1-\sin^2\theta\right)}{1+\sin\theta}$

= $\frac{1+\sin\theta-1+\sin^2\theta}{1+\sin\theta}$

= $\frac{\sin\theta(1+\sin\theta)}{(1+\sin\theta)}$

= $\sin\theta$

= RHS

Hence Proved

(iii) $S_n = 5n^2 - 3n$

$S_1 = a_1$

$= 5(1)^2 - 3(1)$

$= 5 - 3$

$= 2$

$a_1 = 2$

$S_2 = a_1 + a_2$

$a_1 + a_2 = 5(1)^2 - 3(2)$

$= 20 - 6$

$= 14$

$\therefore a_1 + a_2 = 14$

$2 + a_2 = 14$

$a_2 = 14 - 2$

$a_2 = 12$

Again, $S_3 = a_1 + a_2 + a_3$

$a_1 + a_2 + a_3 = 5(3)^2 - 3(3)$

$= 45 - 9$

$= 36$

$a_1 + a_2 + a_3 = 36$

$14 + a_3 = 36$

$a_3 = 36 - 14$

$a_3 = 22$

$a_1 = 2, a_2 = 12, a_3 = 22$

Hence square becomes 2, 12, 22, ...

$a_1 = a = 2$

$d = 12 - 2 = 10$

$a_{16} = a + (16 - 1)d$

$= 2 + 15 \times 10$

$= 2 + 150$

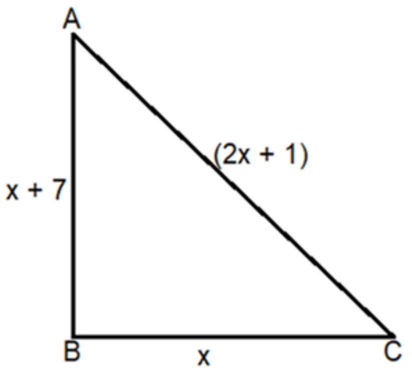
$a_{16} = 152$

7. Question 7

(i) Let the shortest side be x m.

hypotenuse = (2x + 1)m

3rd side = (x + 7)m



$(2x + 1)^2 = (x + 7)^2 + x^2$ {by Pythagoas theorem}

$\Rightarrow 4x^2 + 1 + 4x = x^2 + 49 + 14x + x^2$

$\Rightarrow 2x^2 - 10x - 48 = 0$

$\Rightarrow x^2 - 5x - 24 = 0$

$\Rightarrow x^2 - 8x + 3x - 24 = 0$

$\Rightarrow x(x - 8) + 3(x - 8) = 0$

$\Rightarrow (x + 3)(x - 8) = 0$

x = -3, 8

x = -3 rejected ($\because$ length can never be -ve)

$\therefore$ x = 8

hypotenuse i.e. AC

= 2x + 1

$= 2 \times 8 + 1 = 17$

BC = x = 8m

AB = x + 7

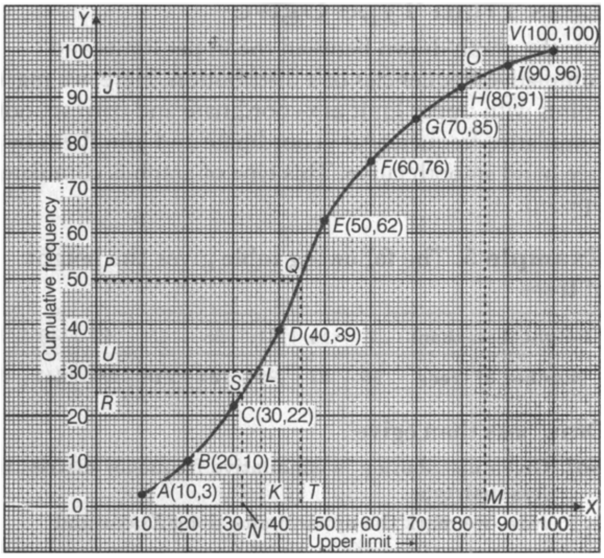
= 8 + 7

AB = 15 m

(ii) The cumulative frequency table for the given continuous distribution is given below

Marks	Number of students	Cumulative frequency (cf)
0-10	3	3
10-20	7	10
20-30	12	22
30-40	17	39
40-50	23	62
50-60	14	76
60-70	9	85
70-80	6	91
80-90	5	96
90-100	4	100

On the graph paper, we plot the following points A (10, 3), B (20, 10), C (30, 22), D (40, 39), E (50, 62), F (60, 76), G (70, 85), H (80, 91), 7(90, 96) and V(100, 100). Join all these points by a free hand drawing. The required ogive is shown on the graph paper given below



Here, number of students (n) = 100, which is even.

i. Let P be the point on Y-axis representing frequency
$= \frac{n}{2} = \frac{100}{2}$ = 50
Through P, draw a horizontal line to meet the ogive at point Q. Through Q, draw a vertical line to meet the X-axis at T. The abscissa of the point T represents 43 marks. Hence, the median marks is 43.

ii. Let R be the point on Y-axis representing frequency

$= \frac{n}{4} = \frac{100}{4}$ = 25.

Through R, draw a horizontal line to meet the ogive at point S. Through S, draw a vertical line to meet the X-axis at N. The abscissa of the point N represents 31 marks. Hence, the lower quartile = 31 marks.

iii. 85% marks = 85% of 100 = 85 marks.

Let the point M on X-axis represents 85 marks. Through M, draw a vertical line to meet the ogive at the point O. Through O draw a horizontal line to meet the Y-axis at point J. The ordinate of point J represents 95 students.

$\therefore$ Number of students who obtained more than 85% in the test = 100 - 95 = 5

iv. 35% marks = 35% of 100 = 35

Let the point K on X-axis represents 35 marks. Through K, draw a vertical line to meet the ogive at the point L. Through L, draw a horizontal line to meet the Y-axis at point U. The ordinate of point U represents 30 students on Y-axis. Hence, the number of students, who did not pass in the test is 30.

8. Question 8

(i) n(s) = 50

n(multiple of 3 and 4) = {12, 24, 36, 48}

i.e multiple of 12

(multiple of 3 and 4) = $\frac{4}{50} = \frac{2}{25}$

(ii) Given, diameter of solid sphere, d_1 = 6 cm

$\therefore$ Radius of sphere, $r_1 = \frac{6}{2}$ = 3 cm

Also, given diameter of cylinder, d_2 = 4 cm

$\therefore$ Radius of cylinder, $r_2 = \frac{4}{2}$ = 2 cm

$\because$ Height of cylinder, h = 45 cm [given]

Let the required number of spheres be N.

$\therefore$ N $\times$ Volume of sphere = Volume of cylinder

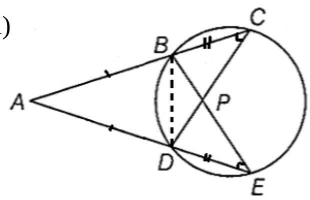
$\Rightarrow N \times \frac{4}{3}\pi r_1^3 = \pi r_2^2$ h

$\Rightarrow N \times \frac{4}{3}\pi \times (3)^3 = \pi \times (2)^2 \times 45$

$\therefore N = \frac{2\times2\times45}{4\times3\times3} = 5$

Hence, the required number of solid spheres is 5.

(iii)



In $\triangle$ADC and $\triangle$ABE

$\angle$ACD = $\angle$AEB (angle in the same segment BD)

AC = AE (given)

$\angle$A = $\angle$A (common)

$\therefore \triangle$ADC $\cong \triangle$ ABE (ASA Còng rule)

$\Rightarrow$ AB = AD (CPCT)

But AC = AE

$\therefore$ AC - AB = AE - AD

$\Rightarrow$ BC = DE

In $\triangle$BPC and $\triangle$DPE

$\angle$C = $\angle$E (angle in the same segment)

BC = DE

$\angle$CBP = $\angle$CDE (angle on the same segment)

$\therefore \triangle$BPC $\cong \triangle$DPE (ASA cong rule)

$\Rightarrow$ BP = DP and CP = PE (CPCT)

9. Question 9

(i) $\frac{3x}{5}$ + 2 < x + 4

$\Rightarrow \frac{3x+10}{5}$ < x + 4

$\Rightarrow$ 3x + 10 < 5(x + 4)

$\Rightarrow$ 3x + 10 < 5x + 20

$\Rightarrow$ 10 - 20 < 5x - 3x

$\Rightarrow$ -10 < 2x

$\Rightarrow$ -5 < x

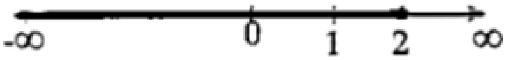
And

x + 4 $\leq \frac{x}{2}$ + 5

$\Rightarrow$ x + 4 $\leq \frac{x+10}{2}$

$\Rightarrow$ 2x + 8 $\leq$ x + 10

$\Rightarrow$ x $\leq$ 2



Solution Set = {x: - 5 < x $\leq$ 2, x $\in$ R}

(ii) Given:

mode = 12 k, mean = 15 k, median = ?

Using Empirical relation;

Mode = 3 median - 2 mean

12k = 3 median - 2(15 k)

3 median = 12k + 30k

3 median = 42k

median = $\frac{42k}{3}$

median = 14k

(iii) Given, DE || BC, DE = 6 cm and BC = 12 cm.

In $\triangle$ABC and $\triangle$ADE,

$\angle$ABC = $\angle$ADE [corresponding angles]

$\angle$ACB = $\angle$AED [corresponding angles]

and $\angle$A = $\angle$A [common angle]

$\therefore \triangle ABC \sim \triangle ADE$ [by AAA similarity criterion]

We know that the ratio of areas of two similar triangles is equal to the ratio of squares of their corresponding sides.

$\therefore \frac{\operatorname{ar}(\Delta ADE)}{\operatorname{ar}(\triangle ABC)}=\frac{(DE)^2}{(BC)^2}=\frac{(6)^2}{(12)^2}=\left(\frac{1}{2}\right)^2$

$\Rightarrow \frac{\operatorname{ar}(\triangle ADE)}{\operatorname{ar}(\triangle ABC)}=\frac{1}{4}$

Let ar ($\triangle$ADE) = k, then ar ($\triangle$ABC) = 4k

Now, ar (DECB) = ar ($\triangle$ABC) - ar ($\triangle$ADE)

= 4k - k = 3k

$\therefore$ Required ratio = ar ($\triangle$ADE) : ar (DECB)

= k : 3k = 1 : 3

10. Question 10

(i) Let fourth proportional be x.

Then, (a^3 + 8) : (a^4 - $2a^3$ + $4a^2$) :: (a^2 - 4) : x

$\Rightarrow \frac{a^3+8}{a^4-2a^3+4a^2} = \frac{a^2-4}{x}$

$\Rightarrow$ x(a^3 + 8) = (a^2- 4) (a^4 - $2a^3$ + $4a^2$) [by cross-multiplication]

$\therefore$ x = $\frac{\left(a^2-2^2\right)\times a^2\left(a^2-2a+4\right)}{\left(a^3+2^3\right)}$

= $\frac{a^2(a-2)(a+2)\left(a^2-2a+4\right)}{(a+2)\left(a^2-2a+4\right)}$ = a^2(a - 2)

Hence, the required value of fourth proportional is a^2(a - 2).

(ii) i. Draw a line BC = 4.2 cm.

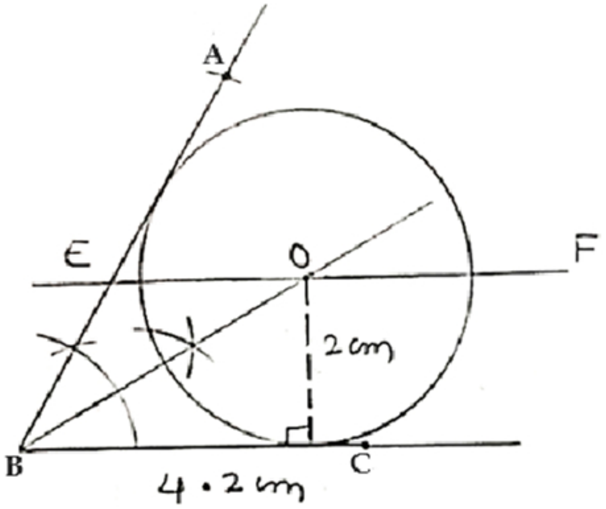
ii. At B, draw an angle of 60° using compass.

iii. Cut an arc of 5 cm on the angle arm at B and name this point as A.

iv. Draw angle bisector of angle ABC.

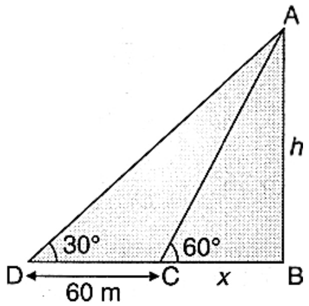
v. Draw a line EFIIBC at a distance of 2 cm which cuts the angle bisector at O.

vi. Take O as centre and 2 cm as radius, draw a circle which touches both the arms.of the angle.



(iii)Let the height of the building be h m and D be the position of a man.

Here, BC = x m



Now, In $\triangle$ABC,

tan 60^o = $\frac{AB}{BC}$

$\sqrt{3} = \frac{h}{x}$

x = $\frac{h}{\sqrt{3}}$ ...(i)

In $\triangle$ABD,

tan 30^o = $\frac{AB}{BD}$

$\frac{1}{\sqrt{3}} = \frac{h}{60+x}$

60 + x = $h\sqrt{3}$

$\sqrt{3}h$ = 60 + $\frac{h}{\sqrt{3}}$ [From eq. (i)]

$\frac{\sqrt{3}h}{1} - \frac{h}{\sqrt{3}}$ = 60

$\frac{3h-h}{\sqrt{3}}$ = 60

2h = $60\sqrt{3}$

h = $\frac{60\sqrt{3}}{2}$

$\Rightarrow$ h = $30\sqrt{3}$

= 30 $\times$ 1.732

Height of building = 51.96 m = 52 m (Approx).

ICSE 2025 EXAMINATION

Sample Question Paper - 3

Mathematics

Time Allowed: 2 hours and 30 minutes **Maximum Marks: 80**

General Instructions:

- Answers to this Paper must be written on the paper provided separately.
- You will not be allowed to write during the first 15 minutes.
- This time is to be spent reading the question paper.
- The time given at the head of this Paper is the time allowed for writing the answers.
- **Attempt all questions from Section A and any four questions from Section B**.
- All work, including rough work, must be clearly shown and must be done on the same sheet as the rest of the answers.
- Omission of essential work will result in a loss of marks.
- The intended marks for questions or parts of questions are given in brackets [ ]
- Mathematical tables are provided.

Section A

1. **Question 1 Choose the correct answers to the questions from the given options:** **[15]**

(a) The marked price of a micro oven is ₹ 10,000. Dealer offers 20% discount on the marked price. The selling price of micro oven: **[1]**

a) ₹ 10,000 b) ₹ 6000
c) ₹ 2000 d) ₹ 8000

(b) The linear factors of the equation $x^2 + kx + 1 = 0$ exists, if **[1]**

a) Both $k \geq 2$ and $k \leq -2$ b) $k \geq 2$
c) $k = 2$ d) $k \leq -2$

(c) When $6x^3 + 2x^2 - x + 2$ is divided by $(x + 2)$, then remainder is **[1]**

a) -36 b) 56
c) 44 d) 36

(d) If both A + B and AB are defined, then which one of the following is true? **[1]**

a) A and B are square matrices of same order b) A and B are square matrices of different order
c) A and B are rectangular matrices of same order d) A and B are rectangular matrices of different order

(e) The sum of series 1 + (1 + 2) + (1 + 2 + 3) + (1 + 2 + 3 + 4)+ ... + (1 + 2 + 3 + ... + 20) is **[1]**

a) 1470 b) 1610

c) 1370 d) 1540

(f) Which of the following points is invariant with respect to the line y = -2? **[1]**

a) (2, 3) b) (3, 2)

c) (-2, 3) d) (3, -2)

(g) In a $\triangle$PQR, L and M are two points on base QR, such that $\angle$LPQ = $\angle$QRP and $\angle$RPM = $\angle$RQP. Then which of the following is/are true **[1]**

i. $\triangle PQL \sim \triangle RPM$

ii. QL $\times$ RM = PL $\times$ PM

iii. PQ^2 = QR·QL

a) All of these b) Both (i) and (iii)

c) Both (i) and (ii) d) Both (ii) and (iii)

(h) A hollow cone of radius 6 cm and height 8 cm is vertical standing at the origin, such that the vertex of the cone is at the origin. Some pipes are hanging around the circular base of the cone, such that they touch the surface of the graph paper. Then, the total surface area of the formed by the figure will be **[1]**

a) 494.68 cm^2 b) 484.98 cm^2

c) 489.84 cm^2 d) 948.84 cm^2

(i) Solve for x : $|x+1|+|x|>3$. **[1]**

a) x $\in (-2, \infty) \cup (-1, \infty)$ b) x $\in (-\infty, -2] \cup [1, \infty)$

c) x $\in (-\infty, -2) \cup (1, \infty)$ d) x $\in [-2, \infty) \cup [-1, \infty)$

(j) The circumcentre of a triangle is the point which is: **[1]**

a) at equal distance from the three sides of the triangle. b) the point of intersection of the three altitudes of the triangle

c) the point of intersection of the three medians. d) at equal distance from the three vertices of the triangle.

(k) If α and β are the roots of the equation x^2 + x - 6 = 0 such that $\beta > \alpha$, then the product of the matrices $\begin{bmatrix} 0 & \alpha \\ \alpha & \beta \end{bmatrix}$ and $\begin{bmatrix} \beta+1 & 0 \\ -\beta & \alpha \end{bmatrix}$ is **[1]**

a) $\begin{bmatrix} -5 & 4 \\ -9 & -2 \end{bmatrix}$ b) $\begin{bmatrix} 6 & 9 \\ -13 & -6 \end{bmatrix}$

c) $\begin{bmatrix} 5 & 4 \\ 9 & 2 \end{bmatrix}$ d) $\begin{bmatrix} 6 & 13 \\ 9 & 6 \end{bmatrix}$

(l) Suppose there are four points A(2, 4), B(6, 4), C(6, 6) and D(2, 6), which lie in the first quadrant. If we rotate only the axes at an angle of 90^{o} in anti-clockwise direction, then what will be the new coordinates of the point C and what will be the name of the figure, when we join adjacent points. **[1]**

a) (6, - 6); rectangle b) (6, 4); square

c) (-6, 6); square

d) (2, -6); rectangle

(m) If P, Q, S and R are points on the circumference of a circle of radius r, such that PQR is an equilateral triangle and PS is a diameter of the circle. Then, the perimeter of the quadrilateral PQSR will be **[1]**

a) $2(\sqrt{3} + 1)r$

b) $2\sqrt{3} + r$

c) 2r

d) $2\sqrt{3}r$

(n) In a colony, the average age of the boys is 14 yr and the average age of the girls is 17 yr. If the average age of the children in the colony is 15 yr, then the ratio of number of boys to that of girls is **[1]**

a) 1 : 1

b) 1 : 2

c) 3 : 2

d) 2 : 1

(o) **Assertion (A):** Three consecutive terms 2k + 1, 3k + 3 and 5k - 1 form an AP than k is equal to 6. **[1]**
Reason (R): In an AP a, a + d, a + 2d, ...the sum of n terms of the AP be $S_n = \frac{n}{2}(2a + (n-1)d)$

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

2. **Question 2** **[12]**

(a) Rashmi has a 4 yr Recurring Deposit Account in Bank of Maharashtra and deposits ₹ 800 per month. If she gets ₹ 48200 at the time of maturity, then find **[4]**

i. the rate of (simple) interest.
ii. the total interest earned by Rashmi.

(b) Mr. Kamal reduces the number of workers of his factory in the ratio 9 : 7 and increases their wages in the ratio 13 : 20. In what ratio, the wages bill is increased or decreased? **[4]**

(c) Prove that: **[4]**

(1 + cot A - cosec A)(1+ tan A + sec A) = 2

3. **Question 3** **[13]**

(a) Circumference of the base of a cylinder, open at the top, is 132 cm. The sum of radius and height is 41 cm. Find the cost of polishing the outer surface area of cylinder at the rate ₹ 10 per sq dm (decimetre). [Take $\pi = \frac{22}{7}$] **[4]**

(b) Find the equation of the line, which passes through the point (3, 4) and the sum of its intercepts on the axes is 14. **[4]**

(c) Using graph paper and taking 1 cm = 1 unit along with X-axis and Y-axis. **[5]**

i. Plot the point A (-4, 4) and B(2, 2).
ii. Reflect A and B in the origin to get the images A' and B', respectively.
iii. Write down the coordinates of A' and B'.
iv. Give the geometrical name for the figure ABA'B'.

Section B

Attempt any 4 questions

4. **Question 4** **[10]**

(a) Let Vinod, Govind and Ankit be three dealers belonging to difference states. Dealer Vinod sells some products/services to dealer Govind for ₹1000 and dealer Govind sells the same products/services to dealer Ankit at a profit of ₹300. Calculate the tax liability of Govind, if the rate of GST is 12%. **[3]**

(b) If the roots of the equation (b - c)x^2 + (c - a)x + (a - b) = 0 are equal, then prove that 2b = a + c. **[3]**

(c) The mean of the following distribution is 52 and the frequency of class-interval 30-40 is f. Find f. **[4]**

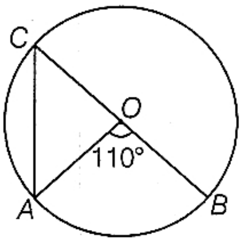
Class-interval	10-20	20-30	30-40	40-50	50-60	60-70	70-80
Frequency	5	3	f	7	2	6	13

Find the value of f.

5. **Question 5** **[10]**

(a) If $\begin{bmatrix} x-y & 2x+z \\ 2x-y & 3z+w \end{bmatrix} = \begin{bmatrix} -1 & 5 \\ 0 & 13 \end{bmatrix}$, then find the values of x, y, z and w. **[3]**

(b) In the given figure, O is the centre of circle and ∠AOB = 110°. Calculate **[3]**



i. ∠ACO

ii. ∠CAO.

(c) If one zero of the polynomial $2x^2$ - 5x - (2k + 1) is twice the other, then find both the zeroes of the polynomial and the value of k. **[4]**

6. **Question 6** **[10]**

(a) Find the value of p , if (p, - 2), (-5, 6) and (1, 2) are collinear. **[3]**

(b) Find the value of $\frac{\sin^2 22^\circ + \sin^2 68^\circ}{\cos^2 22^\circ + \cos^2 68^\circ}$ + $\sin^2 63^\circ$ + cos 63° sin 27° **[3]**

(c) If the pth, qth and rth terms of a GP are a, b and c respectively. Prove that $a^{q-r} b^{r-p} c^{p-q}$ = 1. **[4]**

7. **Question 7** **[10]**

(a) If - 5 is a root of the quadratic equation $2x^2$ + px -15 = 0 and the quadratic equation p(x^2 + x) + k = 0 has equal roots, then find the value of k. **[5]**

(b) Marks obtained by 200 students in an examination are given below: **[5]**

Marks	Number of students
0 - 10	5
10 - 20	11
20 - 30	10
30 - 40	20
40 - 50	28
50 - 60	37
60 - 70	40
70 - 80	29
80 - 90	14
90 - 100	6

Draw an ogive for the given distribution taking 2 cm = 10 marks on one axis and 2 cm = 20 students on the other axis.

i. the median marks.
ii. the number of students who failed, if minimum marks required to pass is 40.
iii. if scoring 85 and more marks is considered as grade one, find the number of students who secured grade one in the examination.

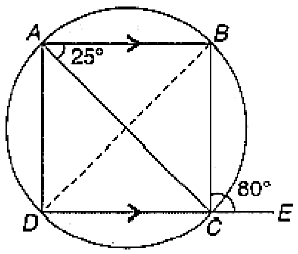
8. **Question 8** **[10]**

(a) A number is chosen from 1 to 100. Find the probability that it is a prime number. **[3]**

(b) The internal and external diameters of steel pipe of length 140 cm are 8 cm and 10 cm, respectively. Then, find the volume of steel. **[3]**

(c) In the figure, AB is parallel to DC, ∠BCE = 80° and ∠BAC = 25°. Find: **[4]**

i. ∠CAD
ii. ∠CBD
iii. ∠ADC



9. **Question 9** **[10]**

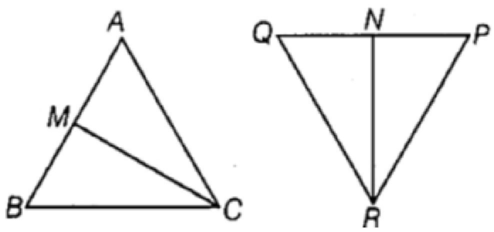
(a) Solve the following inequation and graph the solution set on the number line $x+\frac{2}{15} \leq \frac{-8}{15}, x \in R.$ **[3]**

(b) The following table shows the expenditure of 60 boys on books. **[3]**

Expenditure (in ₹)	20-25	25-30	30-35	35-40	40-45	45-50
No. of students	4	7	23	18	6	2

Find the mode of their expenditure.

(c) In the following figure, CM and RN are respectively the medians of △ABC and △PQR. If $\triangle ABC \sim \triangle PQR$, prove that **[4]**



i. $\triangle AMC \sim \triangle PNR$
ii. $\frac{CM}{RN}=\frac{AB}{PQ}$

10. **Question 10** **[10]**

(a) Two positive numbers are in the ratio 3 : 5 and the difference between their squares is 400. Find the numbers. **[3]**

(b) Draw a circle of radius 4 cm. Mark a point A outside the circle. Draw the tangents to the circle from point A, without using the centre of the circle. **[3]**

(c) An aeroplane at an altitude of 1500 metres finds that two ships are sailing towards it in the same direction. The angles of depression as observed from the aeroplane are 45^{o} and 30^{o} respectively. Find **[4]**

the distance between the two ships.

Solution

Section A

1. Question 1 Choose the correct answers to the questions from the given options:

(i) **(d)** ₹ 8000

Explanation: {

Discount = 20% of ₹ 10,000 = (20 $\div$ 100) $\times$ ₹ 10,000 = ₹ 2000

Selling price = ₹(10,000 – 2000) = ₹ 8,000

(ii) **(a)** Both k $\geq$ 2 and k $\leq$ -2

Explanation: {

We have, $x^2 + kx + 1 = 0$

On comparing it with $ax^2 + bx + c = 0$, we get

a = 1, b = k and c = 1

For the linear factors, D $\geq$ 0

$\Rightarrow b^2 - 4ac \geq 0$

$\Rightarrow c^2 - 4 \times 1 \times 1 \geq 0$

$\Rightarrow (k^2 - 2^2) \geq 0$

$\Rightarrow (k - 2)(k + 2) \geq 0$

$\therefore$ k $\geq$ 2 or k $\leq$ - 2

(iii) **(a)** -36

Explanation: {

Let $f(x) = 6x^3 + 2x^2 - x + 2$

Remainder = $f(-2) = 6(-2)^3 + 2(-2)^2 - (-2) + 2$

$= 6(-8) + 2 \times 4 + 2 + 2$

$= -36$

(iv) **(a)** A and B are square matrices of same order

Explanation: {

Since, A + B is defined, therefore both A and B are of the same type.

Suppose that both A and B are of order m $\times$ n.

Also, AB is defined.

Thus, the number of columns in the pre-factor A must be equal to the number of rows in the post-factor B, i.e. n = m

Hence, both A and B are of order n $\times$ n, i.e. A and B are square matrices of the same type.

(v) **(d)** 1540

Explanation: {

Let S = 1 + (1 + 2) + (1 + 2 + 3) + (1 + 2 + 3 + 4) + ... + (1 + 2 + 3 + ... + 20)

= 1 + 3 + 6 + 10 + 15 + 21 + 28 + 36 + 45 + 55 + 66 + 78 + 91 + 105 + 120 + 136 + 153 + 171 + 190 + 210

= 1540 [$\because 1 + 2 + 3 + ... + n = \frac{n(n+1)}{2}$]

(vi) **(d)** (3, -2)

Explanation: {

We know that a point is invariant when the line of reflection passing through it.

$\because$ (3, -2) lie on y = -2.

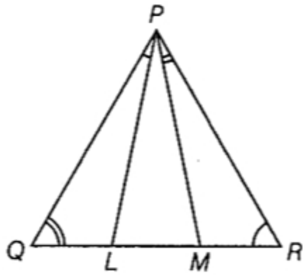
$\therefore$ (3, -2) is the required point.

(vii) **(a)** All of these

Explanation: {

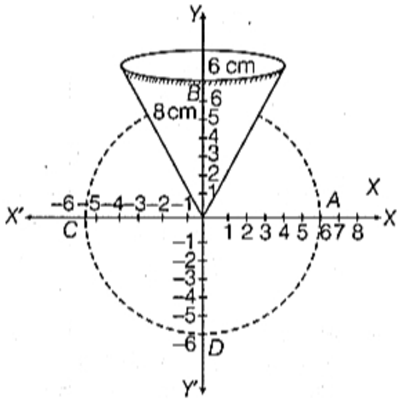
Given $\angle LPQ = \angle QRP$

and $\angle RPM = \angle RQP$



In $\triangle$PQL and $\triangle$RPM, $\angle$LPQ = $\angle$MRP [$\because$ $\angle$LPQ = $\angle$QRP, given]
and $\angle$LQP = $\angle$RPM [$\because$ $\angle$RQP = $\angle$RPM, given]
$\therefore \triangle PQL \sim \triangle RPM$ [by AA similarity criterion]
Since, $\triangle PQL \sim \triangle RPM$
$\therefore \frac{QL}{PM} = \frac{PL}{RM}$ $\Rightarrow$ QL $\times$ RM = PL $\times$ PM
In $\triangle$PQL and $\times$RQP,
$\angle$PQL = $\angle$RQP [common angle]
and $\angle$QPL = $\angle$QRP [given]
$\therefore \triangle PQL \sim \triangle RQP$ [by AA similarity criterion]
Then, $\frac{PQ}{QR} = \frac{QL}{PQ}$
$\Rightarrow$ PQ^2 = QR $\times$ QL

(viii) **(c)** 489.84 cm^2
Explanation: {
According to the given information, a shape of figure is shown below



When the hanging pipes touches the surface paper, a circular shape ABCD is formed on the graph paper. The size of circle ABCD is equal to the size of circular base of the cone.
$\therefore$ Radius of the circle ABCD is 6 cm.
Hence, the coordinates of A, B, C and Dare (6, 0), (0, 6), (-6, 0) and (0, -6), respectively.
The figure formed in the given information is cylindrical in outer surface and conical in the inner surface. Now, total surface area of the figure
= Curved surface area of the cylinder + Curved surface area of the cone
= 2πrh + πrl = πr (2h + l)
= πr(2h + $\sqrt{r^2 + h^2}$)
= 3.14 $\times$ 6$\left(2 \times 8 + \sqrt{6^2 + 8^2}\right)$
= 18.84(16 + $\sqrt{36 + 64}$)
= 18.84(16 + $\sqrt{100}$) = 18.84(16 + 10)
= 18.84 $\times$ 26 = 489.84 cm^2

(ix) **(c)** x $\in$ $(-\infty, -2) \cup (1, \infty)$
Explanation: {
Given inequation is |x + 1| + |x| > 3
Put x + 1 = 0 $\Rightarrow$ x = -1 and x = 0.
So, we will consider three intervals $(-\infty, -1), (-1, 0)$ and $(0, \infty)$.
Case I When $-\infty$ < x < -1, then |x + 1| = -(x + 1) and |x| = -x.

$\therefore$ |x + 1| + |x| > 3 $\Rightarrow$ -(x + 1) -x > 3

$\Rightarrow$ -x - 1 - x > 3 $\Rightarrow$ -2x - 1 > 3

$\Rightarrow$ -2x - 1 + 1 > 3 + 1 [adding 1 both sides]

$\Rightarrow$ -2x > 4 $\Rightarrow$ $\frac{-2x}{-2} < \frac{4}{-2}$ [dividing both sides by -2]

$\Rightarrow$ x < -2

Case II When -1 $\leq$ x < 0, then |x + 1| = x + 1 and |x| = -x.

$\therefore$ |x + 1| + |x| > 3

$\Rightarrow$ x + 1 - x > 3

$\Rightarrow$ 1 > 3, which is not possible.

Case III When 0 $\leq$ x < ∞, then |x + 1| = x + 1 and |x| = x.

$\therefore$ |x + 1| + |x| > 3 $\Rightarrow$ x + 1 + x > 3

$\Rightarrow$ 2x + 1 > 3 $\Rightarrow$ 2x + 1 - 1 > 3 - 1 [subtracting 1 from both sides]

$\Rightarrow$ 2x > 2 $\Rightarrow$ $\frac{2x}{2} > \frac{2}{2}$ [dividing both sides by 2]

$\Rightarrow$ x > 1

On combining results of the above cases, we get

x < -2 or x > 1

$\therefore$ x $\in (-\infty, -2) \cup (1, \infty)$

(x) **(d)** at equal distance from the three vertices of the triangle.

Explanation: {

Circumcenter of a triangle is a point equidistant from all the vertices of the triangle

(xi) **(b)** $\begin{bmatrix} 6 & 9 \\ -13 & -6 \end{bmatrix}$

Explanation: {

Given quadratic equation is x^2 + x - 6 = 0

$\Rightarrow$ x^2 + 3x - 2x - 6 = 0 [by splitting the middle term]

$\Rightarrow$ x(x + 3) - 2(x + 3) = 0

$\Rightarrow$ (x + 3) (x - 2) = 0

$\Rightarrow$ x = -3, 2

Also, given $\beta > \alpha$

$\therefore$ We take β = 2 and α = -3

Now, $\begin{bmatrix} 0 & \alpha \\ \alpha & \beta \end{bmatrix} \begin{bmatrix} \beta+1 & 0 \\ -\beta & \alpha \end{bmatrix} = \begin{bmatrix} 0-\alpha\beta & 0+\alpha^2 \\ \alpha\beta+\alpha-\beta^2 & 0+\alpha\beta \end{bmatrix}$

$= \begin{bmatrix} -(-3)(2) & (-3)^2 \\ (-3)(2)-3-(2)^2 & (-3)(2) \end{bmatrix}$

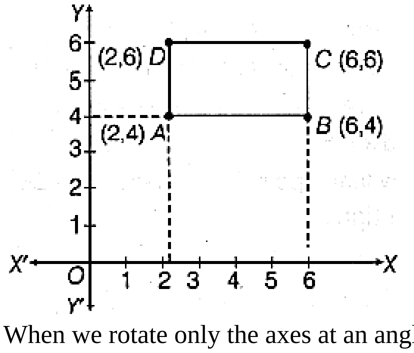
$= \begin{bmatrix} 6 & 9 \\ -6-3-4 & -6 \end{bmatrix}$

$= \begin{bmatrix} 6 & 9 \\ -13 & -6 \end{bmatrix}$

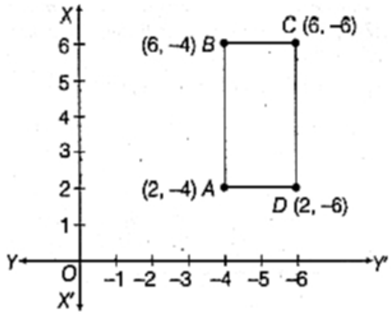
(xii) **(a)** (6, - 6); rectangle

Explanation: {

Given points A(2, 4), B(6, 4), C(6, 6) and D(2, 6) plotting on a graph paper, is shown below



When we rotate only the axes at an angle of 90° in anti-clockwise direction, the new axes are shown below



Here, we see that, in first quadrant, y-coordinates will be negative.

$\therefore$ The new coordinates of A, B, C and D are respectively

A(2, - 4), B(6 - 4), C(6, - 6) and D(2, - 6)

Now, AB = $\sqrt{(6-2)^2+(-4+4)^2}$

= $\sqrt{4^2+0^2}$ = 4 units [$\because$ distance = $\sqrt{\left(x_2-x_1\right)^2+\left(y_2-y_1\right)^2}$]

BC = $\sqrt{(6-6)^2+(-6+4)^2}$ = $\sqrt{(0)^2+(-2)^2}$ = 2 units

CD = $\sqrt{(2-6)^2+(-6+6)^2}$ = $\sqrt{(-4)^2+0^2}$ = 4 units

and DA = $\sqrt{(2-2)^2+(-6+4)^2}$ = $\sqrt{0^2+(-2)^2}$ = 2 units

$\therefore$ AB = CD and BC = DA

Now, the diagonals are

AC = $\sqrt{(6-2)^2+(-6+4)^2}$

= $\sqrt{4^2+(-2)^2}$ = $\sqrt{16+4}=\sqrt{20}$

= $2\sqrt{5}$ units

and BD = $\sqrt{(2-6)^2+(-6+4)^2}$

= $\sqrt{(-4)^2+(-2)^2}$ = $\sqrt{16+4}$ = $\sqrt{20}$

= $2\sqrt{5}$ units

$\therefore$ AC = BD

Hence, the adjacent points A B, C and D from a rectangle.

(xiii) **(a)** $2(\sqrt{3}+1)$r

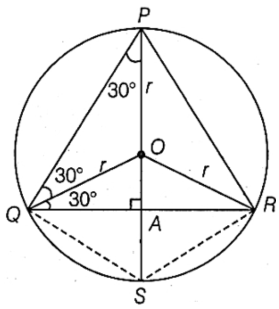
Explanation: {

As PQR is an equilateral triangle, hence PS will be perpendicular to QP and will divide it into 2 equal parts.

Since, $\angle$P and $\angle$S will be supplementary, so

$\angle$S = 120^o and $\angle$QSA = $\angle$RSA = 60^o

Now, PA = PQ cos 30^o and OA = OQ sin 30^o = $\frac{r}{2}$



$\Rightarrow$ AS = OA = $\frac{r}{2}$ and PA = PO + OA = r + $\frac{r}{2}$

Hence, PQ = $\frac{PA}{\cos 30^{\circ}}=\frac{r+\frac{r}{2}}{\frac{\sqrt{3}}{2}}=\sqrt{3}r$

In $\triangle$QAS, AS = QS cos 60^o $\Rightarrow$ QS = $\frac{\frac{r}{2}}{\frac{1}{2}}$ = r

Since, AQ = AR, AS is common and $\angle$QAS = $\angle$RAS = 90^o

So, QS = RS.

$\therefore$ Perimeter of PQSP = 2(PQ + QS) = $2(\sqrt{3}+1)$r

(xiv) **(d)** 2 : 1

Explanation: {

Let n_1 be the number of boys and n_2 be the number of girls.

Then, total ages of boys = $14 \times n_1$

and total ages of girls = $17 \times n_2$

Now, average of children = $\frac{14n_1+17n_2}{n_1+n_2}$

$\Rightarrow 15 = \frac{14n_1+17n_2}{n_1+n_2}$

$\Rightarrow 15n_1 + 15n_2 = 14n_1 + 17n_2$

$\Rightarrow n_1 = 2n_2 \Rightarrow \frac{n_1}{n_2} = \frac{2}{1}$

or $n_1 : n_2 = 2 : 1$

(xv) **(b)** Both A and R are true but R is not the correct explanation of A.

Explanation: {

For 2k + 1, 3k + 3 and 5k - 1 to form an AP

(3k + 3) - (2k + 1) = (5k - 1) - (3k + 3)

k + 2 = 2k - 4

2 + 4 = 2k - k = k

k = 6

So, both assertion and reason are correct but reason does not explain assertion.

2. Question 2

(i) Given

P = ₹ 800/month

n = 4 yr = 48 months

m.v. = 48,200

(i) r = ?, (ii) Total Interest = ?

i. m.v. = $pn + \frac{p \cdot r \cdot n(n+1)}{2400}$

48,200 = $(800 \times 48) + \frac{800 \times 48 \times 49 \times r}{2400}$

48,200 = 38400 + 784 r

r = $\frac{48{,}200 - 38{,}400}{784}$

r = 12.5 p.a.

ii. Total Interest = $\frac{p \cdot r \cdot n(n+1)}{2400}$

$\frac{800 \times 12.5 \times 48 \times 49}{2400}$

Total Interest = ₹ 9800

Hence, Total Interest earned by Rashmi = ₹9800

(ii) Let the total no. of employees initially be 9x.

After reducing the total no. of employees = 7x

and, Wages before reduction = 13y

Wages after reduction = 20y

We know that,

Total Wages = total employees $\times$ wages per employee

$\therefore$ Total wages(before reduction) = 9x $\times$ 13y = 117 xy

Total wages (after reduction) = 7x $\times$ 20y = 140 xy

Ratio of wage bill = 117xy : 140xy = 117 : 140

Hence wage bill ratio increases to 117 : 140.

(iii) LHS = (1 + cot A - cosec A)(1 + tan A + sec A)

$= \left(1 + \frac{\cos A}{\sin A} - \frac{1}{\sin A}\right)\left(1 + \frac{\sin A}{\cos A} + \frac{1}{\cos A}\right)$

$= \left(\frac{\sin A + \cos A - 1}{\sin A}\right)\left(\frac{\sin A + \cos A + 1}{\cos A}\right)$

$= \frac{(\sin A + \cos A)^2 - 1^2}{\sin A \cos A}$ [Using (a - b) (a + b) = $a^2 - b^2$]

$= \frac{\sin^2 A + \cos^2 A + 2\sin A\cos A - 1}{\sin A \cos A}$

$= \frac{1+2\sin A\cos A-1}{\sin A\cos A}$

$= \frac{2\sin A\cos A}{\sin A\cos A}$

= 2 = RHS.

Hence proved

3. Question 3

(i) Circumference of a cylinder = 132

$\Rightarrow 2 \times \frac{22}{7} \times r$ = 132

$\therefore$ r = 21

Given, h + r = 41

$\Rightarrow$ h + 21 = 41

$\therefore$ h = 41 - 21 = 20

Outer Surface Area of Cylinder = $2\pi rh + \pi r^2$

= $\pi r(2h + r)$

$= \frac{22}{7} \times 21(2 \times 20 + 21)$

= 22 $\times$ 3 $\times$ (61)

= 4026 cm^2

= $\frac{4026}{100} cm^2$

= 40.26 cm^2

Total cost = 40.26 $\times$ 10 = ₹402.60

(ii) Let the equation of line in intercept for $\frac{x}{a} + \frac{y}{b}$ = 1

$\because$ It passes through (3, 4)

$\therefore \frac{3}{a} + \frac{4}{b}$ = 1 ...(i)

Also given that a + b = 14

$\Rightarrow$ b = 14 - a

put the value of b is (i) we get.

$\frac{3}{a} + \frac{4}{14-a}$ = 1

3(14 - a) + 4a = a(14 - a)

$\Rightarrow$ 42 - 3a + 4a = 14a - a^2

$\Rightarrow$ 42 + a = 14a - a^2

$\Rightarrow$ a^2 + a - 14a + 42 = 0

$\Rightarrow$ a^2 - 13a + 42 = 0

$\Rightarrow$ a^2 - 7a - 6a + 42 = 0

$\Rightarrow$ a(a - 7) - 6(a - 7) = 0

$\Rightarrow$ (a - 7) (a - 6) = 0

$\Rightarrow$ a = 7 or a = 5

Case 1: If a = 7, b = 14 - 7 = 7

$\therefore$ equation of line

$\frac{x}{7} + \frac{y}{7}$ = 1 $\Rightarrow$ x + y = 7

Case 2: If a = 6, b = 14 - 6 = 8

$\therefore$ equation of line is,

$\frac{x}{6} + \frac{y}{8}$ = 1 $\Rightarrow$ 8x + 6y = 48

$\Rightarrow$ 4x + 3y = 24

Hence equation of line is

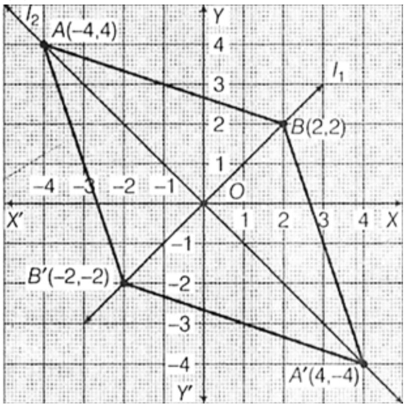
x + y = 7 or 4x + 3y = 24

(iii)(i) (ii) (In the graph paper)

(iii) A'(4, -4), B'(-2, -2)

(iv) Rhombus

(v) AA and B'B'



Section B

4. Question 4

(i) For dealer Vinod,

Selling price = ₹1000 (Given)

$\because$ Since in case of inter-state, we get

IGST = ₹$\frac{12}{100} \times 1000$ = ₹120

For dealer Govind,

cost price = ₹1000

$\therefore$ selling price = ₹(1000 + 300)

= ₹1300

For dealer Ankit,

Cost price = ₹1000

profit = ₹300

Input tax credit = ₹120

output tax = ₹($\frac{12}{100} \times 1300$)

= ₹156

$\therefore$ Tax liability on dealer Govind = ₹(156 - 120)

= ₹36

(ii) $(b - c)x^2 + (c - a)x + (a - b) = 0$

$\because$ since Roots are given equal.

By hit and trial, on putting x = 1, it satisfy the equation

Hence x = 1, 1 are the roots of above equation

$\because$ We know that;

Product of roots = $\frac{C}{A} = \frac{a-b}{b-c}$

Here product of roots = $1 \times 1 = 1$

$\therefore \frac{a-b}{b-c} = 1$

$\Rightarrow$ a - b = b - c

$\Rightarrow$ a + b = 2b

(iii)

CI.	f	Mid Value x	fx
10-20	5	15	75
20-30	3	25	75
30-40	f	35	35f
40-50	7	45	315
50-60	2	55	110
60-70	6	65	390
70-80	13	75	975
Total	36 + f		1940 + 35f

$\bar{x} = \frac{\sum fx}{\Sigma t}$

52 = $\frac{1940+35f}{36+f}$ [$\because \bar{x}$ = 52 given]

52(36 + f) = 1940 + 35 f

1940 + 35 f = 1872 + 52 f

1940 - 1872 = 52 f - 35 f

68 = 17f

f = $\frac{68}{17}$

f = 4

5. Question 5

(i) Given, $\begin{bmatrix} x-y & 2x+z \\ 2x-y & 3z+w \end{bmatrix} = \begin{bmatrix} -1 & 5 \\ 0 & 13 \end{bmatrix}$

Here, both matrices are equal, so to find x, y, z and w, we equate the corresponding elements.

On equating the corresponding elements, we get

x - y = -1 ...(i)

2x + z = 5 ...(ii)

2x - y = 0 ...(iii)

and 3z + w = 13 ...(iv)

On solving Eqs. (i) and (iii), we get

x = 1 and y = 2

On putting the value of x in Eq. (ii), we get

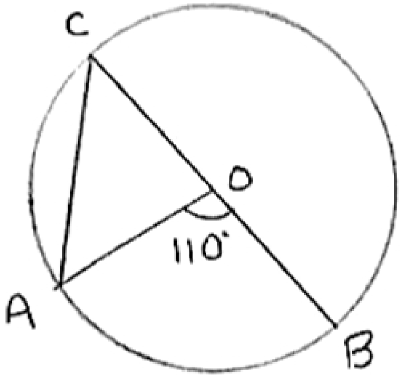
2 $\times$ 1 + z = 5 $\Rightarrow$ z = 5 - 2 = 3

On putting the value of z in Eq. (iv), we.get

3 $\times$ 3 + w = 13 $\Rightarrow$ w = 13 - 9 = 4

Hence, x = 1, y = 2, z = 3 and w = 4.

(ii)



$2\angle ACB = \angle AOB$ (angle made by same arc)

$\angle ACB = \frac{110^\circ}{2}$

$\angle ACB = 55^\circ$

In $\triangle AOC$

AO = OC (radius)

$\angle ACO = \angle CAO$ (angle opposite to equal side of $\triangle$)

$\triangle CAO$ = 55^o

(iii) Let α and 2α are the zeroes of the polynomial $2x^2$ - 5x - (2k + 1).

Then, $2\alpha^2$ - 5α - (2k + 1) = 0

and 2 $(2\alpha)^2$ - $5(2\alpha)$ - (2k + 1) = 0

$\Rightarrow$ $2\alpha^2$ - 5α = 2k + 1 ...(i)

and $8\alpha^2$ - 10α = 2k + 1 ...(ii)

From Eqs. (i) and (ii), we get

$2\alpha^2$ - 5a = $8\alpha^2$ - 10α $\Rightarrow$ $6\alpha^2$ = 5α $\Rightarrow$ $\alpha = \frac{5}{6}$ $[\because \alpha \neq 0]$

$\therefore 2\alpha$ = $\frac{5}{6} \times 2 = \frac{5}{3}$

Thus, the zeroes of the polynomial are $\frac{5}{6}$ and $\frac{5}{3}$

Now, substituting $\alpha = \frac{5}{6}$ in Eq. (i), we get

$2 \times \frac{25}{36} - \frac{25}{6}$ = 2k + 1

$\Rightarrow$ 2k + 1 = $\frac{50-150}{36} \Rightarrow 2k + 1$ = $-\frac{100}{36}$

$\Rightarrow$ 2k = $-\frac{100}{36} - 1 \Rightarrow 2k = -\frac{136}{36}$

$\Rightarrow$ k = - $\frac{68}{36}$ = $\frac{-17}{9}$

6. Question 6

(i) Let three points be A(p, -2), B(-5, 6) and C(1, 2)

$\because$ We know that if ar($\triangle$ABC) = 0 then points are collinear.

ar($\triangle$ABC) = $\frac{1}{2}|x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)|$

$\Rightarrow$ 0 = $\frac{1}{2}$|p(6 - 2) + (-5)(2 + 2) +1(-2 - 6)|

$\Rightarrow$ 0 = $\frac{1}{2}$|4p - 20 - 8|

$\Rightarrow$ 0 = $\frac{1}{2}$|4p - 28|

$\Rightarrow$ 4p - 28 = 0

$\therefore$ p = $\frac{28}{4}$ = 7

Hence, the value of p is 7.

(ii) $\frac{\sin^2 22^\circ + \sin^2 68^\circ}{\cos^2 22^\circ + \cos^2 68^\circ}$ + $\sin^2 63^o + \cos 63^o \sin 27^o$

$\frac{\sin^2 22^o + \sin^2(90-22)}{\cos^2 22^\circ + \cos^2(90-22)}$ + $\sin^2 63^o + \cos 63^o$ sin(90 - 63)

$\frac{\sin^2 22^o + \cos^2 22}{\sin^2 22^\circ + \cos^2 22}$ + $\sin^2 63 + \cos^2 63$

$= \frac{1}{1} + 1$

= 2

(iii) Let first term = A

Common ratio = R

a = pth term = $A \cdot R^{p-1}$

b = qth term = $A \cdot R^{q-1}$

c = rth term = $A \cdot R^{r-1}$

from LHS

$a^{q-r} \cdot b^{r-p} \cdot c^{p-q}$

$\Rightarrow \left(A \cdot R^{p-1}\right)^{q-r} \cdot \left(A \cdot R^{q-1}\right)^{r-P} \cdot \left(A \cdot R^{r-1}\right)^{p-q}$

$\Rightarrow A^{q-r} R^{(p-1)(q-r)} A^{r-p} R^{(q-1)(r-p)} A^{p-q} \cdot R^{(r-1)(p-q)}$

$\Rightarrow (A)^{q-r+r-p+p-q} . R^{(p-1)(q-r)+(q-1)(r-p)+(r-1)(p-q)}$

$\Rightarrow A^0 R^{pq-q-pr+r+qr-r-pq+p+pr-p-qr+q}$

$\Rightarrow A^0 R^0 = 1$

Hence, LHS = RHS

So, $a^{q-r} b^{r-p} c^{p-q} = 1$ **Proved**

7. Question 7

(i) $2x^2 + px - 15 = 0$

$\because$ (-5) is a root of above equation

So it must satisfy the given equation

$\Rightarrow 2(-5)^2 + p(-5) - 15 = 0$

$\Rightarrow$ 50 - 5p - 15 = 0

$\Rightarrow$ -5p = 35

p = 7

$p(x^2 + x) + k = 0$

$\Rightarrow 7(x^2 + x) + k = 0$

$\Rightarrow 7x^2 + 7x + k = 0$

For real and equal roots

D = 0

$\Rightarrow (7)^2 - 4(7)(k) = 0$

$\Rightarrow$ 49 - 28k = 0

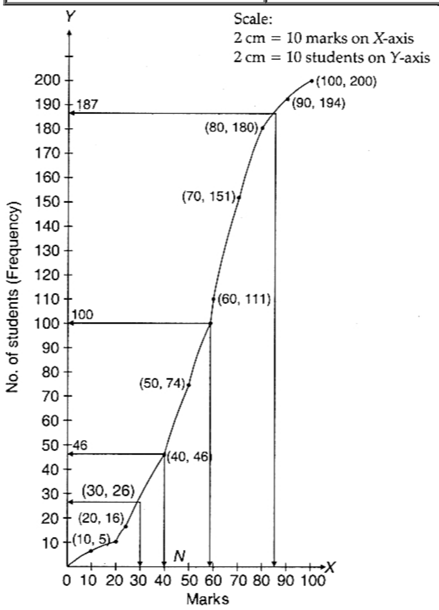
$\Rightarrow 28k = 49$

$\Rightarrow k = \frac{49}{28}$

$\Rightarrow k = \frac{7}{4}$

(ii)

Marks	Frequency (f)	Cumulative frequency (cf)
0 - 10	5	5
10 - 20	11	16
20 - 30	10	26
30 - 40	20	46
40 - 50	28	74
50 - 60	37	111
60 - 70	40	151
70 - 80	29	180
80 - 90	14	194
90 - 100	6	200
	N = 200	



i. Median = $\left(\frac{N}{2}\right)^{th} = \left(\frac{200}{2}\right)^{th}$ term = 57

ii. 46

iii. 200 - 187 = 13

8. Question 8

(i) Total no of possible outcomes = 100

Total prime no between 1 to 100 are {2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 73, 79, 83, 89, 97} i.e. total 25 prime numbers.

P(getting a prime no.) = $\frac{\text{Total prime number}}{\text{Total numbers}} = \frac{25}{100} = \frac{1}{4}$

(ii) External diameter of steel pipe = 10 cm

external radius of steel (R) = $\frac{10}{2}$ = 5 cm

Internal diameter of steel pipe = 8 cm

Internal radius of steel pipe = $\frac{8}{2}$ = 4 cm

Volume of steel = π (R^2 - r^2) h

= $\frac{22}{7}$ (5^2 - 4^2) 140

$= \frac{22}{7}(5 + 4)(5 - 4)\ 140$
$= \frac{22}{7} \times 9 \times 1 \times 140$
$= 3960\ cm^3$

Hence volume of steel be $3960\ cm^3$

(iii) i. $\angle BCE = \angle DAB = 80^o$

$\because$ (External angle of cyclic quadrilateral is equal to opposite interior angle)

$\angle CAD = \angle DAB - \angle BAC$

$= 80 - 25° = 55°$

ii. $\angle CBD = \angle CAD$ (Angles in the same segment)

$= 55^o$

iii. $\angle ABC = \angle BCE$ (Alternate interior $\angle$)

$\therefore \angle ABC = 80°$

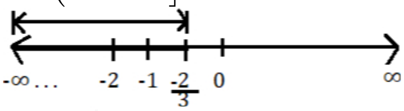
$\angle ADC = 180^o - \angle ABC$ (Opposite angels of cyclic quadrilateral)

$= 180° - 80$

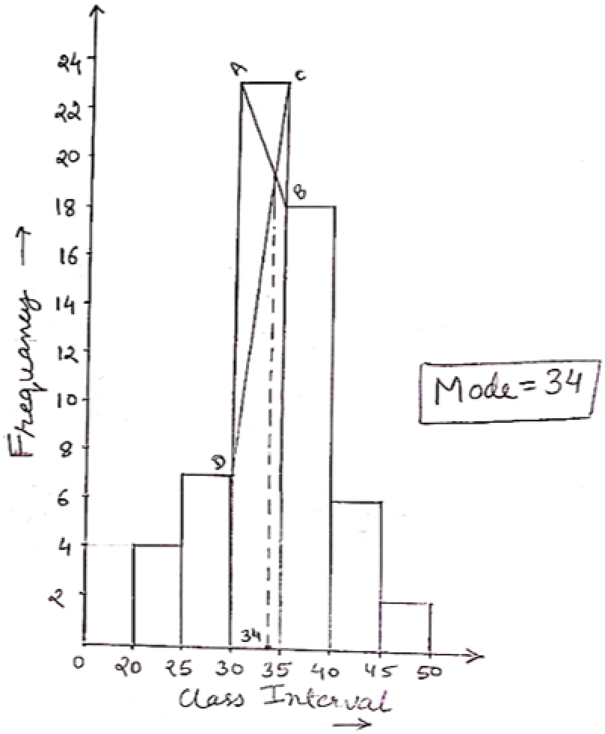
$= 100°$

9. Question 9

(i) $x + \frac{2}{15} \leq \frac{-8}{15}$
$x \leq \frac{-8}{15} - \frac{2}{15}$
$x \leq \frac{-8-2}{15}$
$x \leq \frac{-10}{15}$
$x \leq \frac{-2}{3}$
$x \in \left(-\infty, -\frac{2}{3}\right]$



(ii)



(iii) i. Given, $\triangle ABC \sim \triangle PQR$

$\therefore \frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR}$ ...(i)

and $\angle A = \angle P$, $\angle B = \angle Q$ and $\angle C = \angle R$ ...(ii)

We know that the median bisects the opposite side.

$\therefore$ AM = MB $\Rightarrow$ AB = 2 AM and PN = NQ $\Rightarrow$ PQ = 2PN

From Eq. (i), we have

$\frac{AB}{PQ} = \frac{AC}{PR} \Rightarrow \frac{2AM}{2PN} = \frac{AC}{PR}$

$\Rightarrow \frac{AM}{PN} = \frac{AC}{PR}$ ...(iii)

In $\triangle$AMC and $\triangle$PNR, $\angle$A = $\angle$P [from Eq. (ii)]

and $\frac{AM}{PN} = \frac{AC}{PR}$ [from Eq. (iii)]

So, $\triangle AMC \sim \triangle PNR$ [by SAS similarity criterion]

ii. We have, $\triangle AMC \sim \triangle PNR \Rightarrow \frac{AM}{PN} = \frac{AC}{PR} = \frac{CM}{RN}$ [$\because$ triangles are similar, hence corresponding sides will be proportional]

$\therefore \frac{CM}{RN} = \frac{AC}{PR} \Rightarrow \frac{CM}{RN} = \frac{AM}{PN}$ [from Eq. (iii)]

$\Rightarrow \frac{CM}{RN} = \frac{2AM}{2PN}$ [multiplying numerator and denominator by 2 in RHS]

$\Rightarrow \frac{CM}{RN} = \frac{AB}{PQ}$ [$\because$ AB = 2AM and PQ = 2PN] ...(iv)

10. Question 10

(i) Let the numbers are 3x and 5x respectively.

Then, $(5x)^2 - (3x)^2 = 400$

$\Rightarrow 25x^2 - 9x^2 = 400$

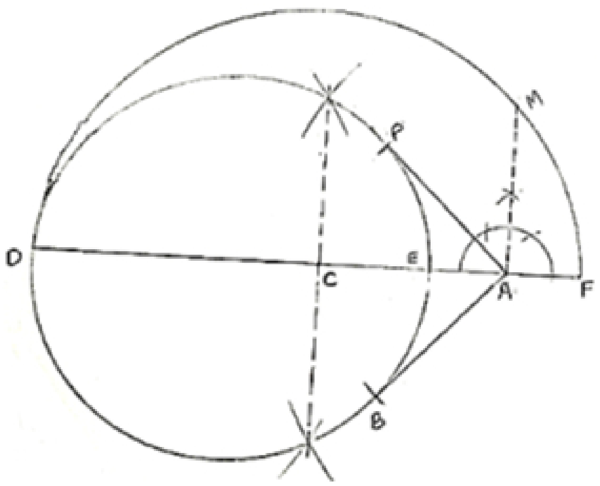
$\Rightarrow 16x^2 = 400$

$x^2 = \frac{400}{16}$ = 25

$\therefore$ x = $\sqrt{25}$ = ± 5

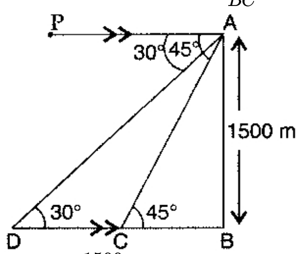
Hence, the numbers are 15 and 25.

(ii)
i. Draw a circle with radius 4cm.
ii. Take any chord DE.
iii. Take a point A outside the circle and join EA.
iv. Take a point F such that EA = AF
v. Draw perpendicular bisector of DF which cuts DF at C.
vi. Take C as centre and DC as radius draw a semicircle.
vii. Draw a perpendicular line on A which cuts semicircle at M.
viii. Take AM as radius draw two arcs which cuts the circle at P and B.
ix. Join AP and AB.
x. AP and AB are the required tangents.



(iii) Let AB be the altitude and C and D be the positions of the two ships, then AB = 1500 m $\angle$PAD = $\angle$ADB and $\angle$PAC = $\angle$ACB (Alternate angles)

In $\triangle$ABC, tan 45^o = $\frac{1500}{BC}$



$\Rightarrow 1 = \frac{1500}{BC}$

$\Rightarrow$ BC = 1500 m

$\therefore$ In $\triangle$ABD,

tan 30^o = $\frac{1500}{BD}$

$\therefore \frac{1}{\sqrt{3}} = \frac{1500}{BD}$

BD = $1500\sqrt{3}$

= 1500 $\times$ 1.732

= 2598 m

Distance between the two ships,

CD = BD - BC

= 2598 - 1500 = 1098 m.

ICSE Board
Class X Mathematics
Board Paper 2018
(Two hours and a half)

Answers to this Paper must be written on the paper provided separately.
You will not be allowed to write during the first 15 minutes.
This time is to be spent in reading the question paper.
The time given at the head of this Paper is the time allowed for writing the answers.

Attempt **all** questions from **Section A** and **any four** questions from **Section B**.
All working, including rough work, must be clearly shown and must be done on the same sheet as the rest of the answer.
Omission of essential working will result in loss of marks.
The intended marks for questions or parts of questions are given in brackets [].
Mathematical tables are provided.

SECTION A (40 Marks)

*Attempt **all** questions from this Section.*

Question 1

(a) Find the value of and 'y' if: [3]

$$2\begin{bmatrix} x & 7 \\ 9 & y-5 \end{bmatrix} + \begin{bmatrix} 6 & -7 \\ 4 & 5 \end{bmatrix} = \begin{bmatrix} 10 & 7 \\ 22 & 15 \end{bmatrix}$$

(b) Sonia had a recurring deposit account in a bank and deposited Rs. 600 per month for $2^1/_2$ years. If the rate of interest was 10% p.a., find the maturity value of this account. [3]

(c) Cards bearing numbers 2, 4, 6, 8, 10, 12, 14, 16, 18 and 20 are kept in a bag. A card is drawn at random from the bag. Find the probability of getting a card which is: [4]

(i) a prime number.
(ii) a number divisible by 4.
(iii) a number that is a multiple of 6.
(iv) an odd number.

Question 2

(a) The circumference of the base of a cylindrical vessel is 132 cm and its height is 25 cm. Find the [3]

(i) radius of the cylinder
(ii) volume of cylinder (use $\pi = \frac{22}{7}$)

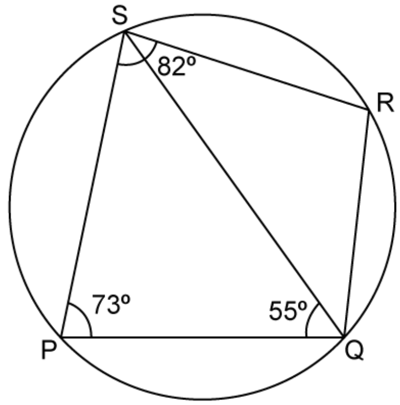
(b) If (k – 3), (2k + l) and (4k + 3) are three consecutive terms of an A.P., find the value of k. [3]

(c) PQRS is a cyclic quadrilateral. Given ∠QPS = 73°, ∠PQS = 55° and ∠PSR = 82°, calculate: [4]

(i) ∠QRS

(ii) ∠RQS

(iii) ∠PRQ



Question 3

(a) If (x + 2) and (x + 3) are factors of $x^3 + ax + b$, find the values of 'a' and `b'. [3]

(b) Prove that $\sqrt{\sec^2\theta+\text{cosec}^2\theta}=\tan\theta+\cot\theta$ [3]

(c) Using a graph paper draw a histogram for the given distribution showing the number of runs scored by 50 batsmen. Estimate the mode of the data: [4]

Runs scored	3000-4000	4000-5000	5000-6000	6000-7000	7000-8000	8000-9000	9000-10000
No. of batsmen	4	18	9	6	7	2	4

Question 4

(a) Solve the following inequation, write down the solution set and represent it on the real number line: [3]

$-2 + 10x \leq 13x + 10 < 24 + 10x, x \in Z$

(b) If the straight lines 3x – 5y = 7 and 4x + ay + 9 = 0 are perpendicular to one another, find the value of a. [3]

(c) Solve $x^2 + 7x = 7$ and give your answer correct to two decimal places. [4]

SECTION B (40 Marks)

*Attempt any **four** questions from this Section*

Question 5

(a) The 4^{th} term of a G.P. is 16 and the 7^{th} term is 128. Find the first term and common ratio of the series. [3]

(b) A man invests Rs. 22,500 in Rs. 50 shares available at 10% discount. If the dividend paid by the company is 12%, calculate: [3]

(i) The number of shares purchased

(ii) The annual dividend received.

(iii) The rate of return he gets on his investment. Give your answer correct to the nearest whole number.

(c) Use graph paper for this question (Take 2 cm = 1 unit along both x and y axis). ABCD is a quadrilateral whose vertices are A(2, 2), B(2, –2), C(0, –1) and D(0, 1). [4]

(i) Reflect quadrilateral ABCD on the y-axis and name it as A'B'CD.

(ii) Write down the coordinates of A' and B'.

(iii) Name two points which are invariant under the above reflection.

(iv) Name the polygon A'B'CD.

Question 6

(a) Using properties of proportion, solve for x. Given that x is positive: [3]

$$\frac{2x+\sqrt{4x^2-1}}{2x-\sqrt{4x^2-1}}=4$$

(b) if $A=\begin{bmatrix}2 & 3\\ 5 & 7\end{bmatrix}, B=\begin{bmatrix}0 & 4\\ -1 & 7\end{bmatrix}$ and $c=\begin{bmatrix}1 & 0\\ -1 & 4\end{bmatrix}$, find $AC + B^2 - 10C$. [3]

(c) Prove that $(1 + \cot\theta - \operatorname{cosec}\theta)(1+ \tan\theta + \sec\theta) = 2$ [4]

Question 7

(a) Find the value of k for which the following equation has equal roots. [3]

$x^2 + 4kx + (k^2 - k + 2) = 0$

(b) On a map drawn to a scale of 1 : 50,000, a rectangular plot of land ABCD has the following dimensions. AB = 6 cm; BC = 8 cm and all angles are right angles. Find: [3]

(i) the actual length of the diagonal distance AC of the plot in km.

(ii) the actual area of the plot in sq. km.

(c) A(2, 5), B(−1, 2) and C(5, 8) are the vertices of a triangle ABC, `M' is a point on AB such that AM : MB = 1 : 2. Find the co-ordinates of 'M'. Hence find the equation of the line passing through the points C and M. [4]

Question 8

(a) Rs. 7500 were divided equally among a certain number of children. Had there been 20 less children, each would have received Rs. 100 more. Find the original number of children. [3]

(b) If the mean of the following distribution is 24, find the value of 'a '. [3]

Marks	0-10	10-20	20-30	30-40	40-50
Number of students	7	a	8	10	5

(c) Using ruler and compass only, construct a ΔABC such that BC = 5 cm and AB = 6.5 cm and ∠ABC = 120° [4]

(i) Construct a circum-circle of ΔABC

(ii) Construct a cyclic quadrilateral ABCD, such that D is equidistant from AB and BC.

Question 9

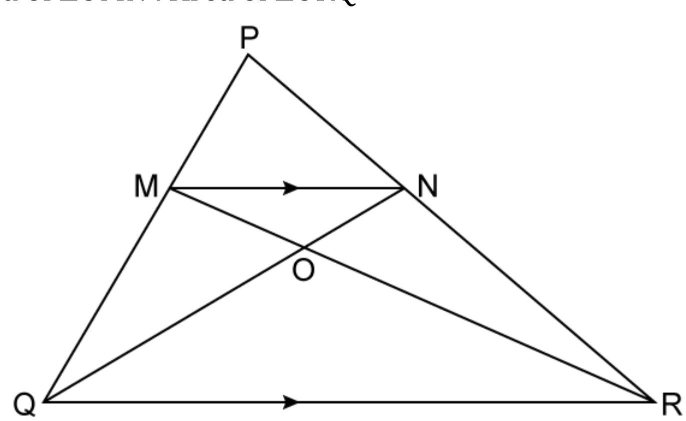
(a) Priyanka has a recurring deposit account of Rs. 1000 per month at 10% per annum. If she gets Rs. 5550 as interest at the time of maturity, find the total time for which the account was held. [3]

(b) In ΔPQR, MN is parrallel to QR and $\frac{PM}{MQ} = \frac{2}{3}$ [3]

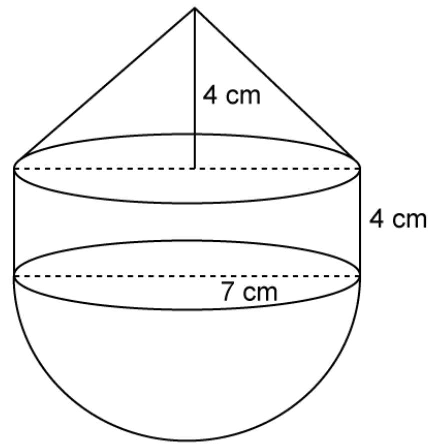
(i) Find $\frac{MN}{QR}$

(ii) Prove that ΔOMN and ΔORQ are similar.

(iii) Find, Area of ΔOMN : Area of ΔORQ



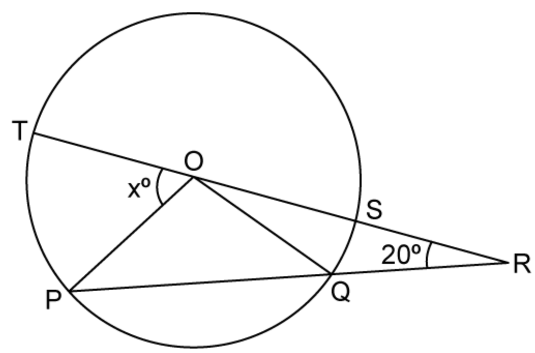
(c) The following figure represents a solid consisting of a right circular cylinder with a hemisphere at one end and a cone at the other. Their common radius is 7 cm. The height of the cylinder and cone are each of 4 cm. Find the volume of the solid. [4]



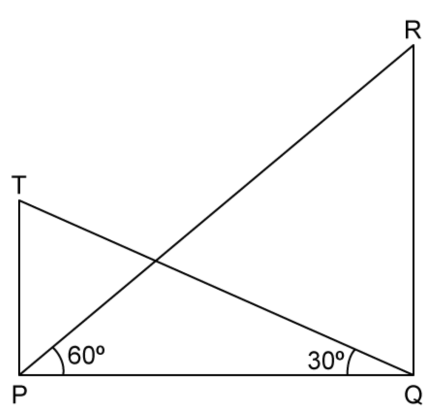
Question 10

(a) Use Remainder theorem to factorize the following polynomial: [3]
$2x^3 + 3x^2 - 9x - 10$.

(b) In the figure given below 'O' is the centre of the circle. If QR = OP and $\angle ORP = 20°$. Find the value of 'x' giving reasons. [3]



(c) The angle of elevation from a point P of the top of a tower QR, 50 m high is $60°$ and that of the tower PT from a point Q is $30°$. Find the height of the tower PT, correct to the nearest metre. [4]



Question 11

(a) The 4^{th} term of an A.P. is 22 and 15^{th} term is 66. Find the first terns and the common difference. Hence find the sum of the series to 8 terms. [4]

(b) Use Graph paper for this question. [6]

A survey regarding height (in cm) of 60 boys belonging to Class 10 of a school was conducted. The following data was recorded:

Height in cm	135-140	140-145	145-150	150-155	155-160	160-165	165-170
No. of boys	4	8	20	14	7	6	1

Taking 2 cm = height of 10 cm along one axis and 2 cm = 10 boys along the other axis draw an ogive of the above distribution. Use the graph to estimate the following:

(i) the median

(ii) lower Quartile

(iii) if above 158 cm is considered as the tall boys of the class. Find the number of boys in the class who are tall.

Solution

SECTION A

1.

(a)

$$2\begin{bmatrix} x & 7 \\ 9 & y-5 \end{bmatrix} + \begin{bmatrix} 6 & -7 \\ 4 & 5 \end{bmatrix} = \begin{bmatrix} 10 & 7 \\ 22 & 15 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2x & 14 \\ 18 & 2y-10 \end{bmatrix} + \begin{bmatrix} 6 & -7 \\ 4 & 5 \end{bmatrix} = \begin{bmatrix} 10 & 7 \\ 22 & 15 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2x+6 & 14-7 \\ 18+4 & 2y-10+5 \end{bmatrix} = \begin{bmatrix} 10 & 7 \\ 22 & 15 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2x+6 & 7 \\ 22 & 2y-5 \end{bmatrix} = \begin{bmatrix} 10 & 7 \\ 22 & 15 \end{bmatrix}$$

$$\Rightarrow 2x+6=10 \quad \text{and} \quad 2y-5=15$$

$$\Rightarrow 2x=4 \quad \text{and} \quad 2y=20$$

$$\Rightarrow x=2 \quad \text{and} \quad y=10$$

(b)

Given : $P = \text{Rs. } 600$, $n = 30$ months and $r = 10\%$

$$\therefore I = \text{Rs.}\left(600 \times \frac{30(30+1)}{2\times 12} \times \frac{10}{100}\right) = \text{Rs. } 2325$$

Since sum deposited $= P \times n = \text{Rs. } 600 \times 30 = \text{Rs. } 18000$

Thus, the maturity value $= \text{Rs. } (18000 + 2325) = \text{Rs. } 20325$

(c)

Total number of cards $= 10$

(i) Prime number card is 2.

$\Rightarrow$ Number of favourable outcomes $= 1$

$\therefore$ Required probability $= \dfrac{1}{10}$

(ii) Cards having number divisible by 4 are 4, 8, 12, 16, 20.

$\Rightarrow$ Number of favourable outcomes $= 5$

$\therefore$ Required probability $= \dfrac{5}{10} = \dfrac{1}{2}$

(iii) Cards having number that is multiple of 6 are 6, 12, 18

$\Rightarrow$ Number of favourable outcomes $= 3$

$\therefore$ Required probability $= \dfrac{3}{10}$

(iv) Odd number card is not there.

$\Rightarrow$ Number of favourable outcomes $= 0$

$\therefore$ Required probability $= 0$

2.

(a)

Let the radius of the cylindrical vessel be r and its height be h.

$\Rightarrow$ Height $= h = 25$ cm

(i) Circumference of the base $= 132$ cm

$\Rightarrow 2\pi r = 132$

$\Rightarrow 2 \times \dfrac{22}{7} \times r = 132$

$\Rightarrow r = 21$ cm

(ii) Volume of cylinder $= \pi r^2 h$

$$= \frac{22}{7} \times 21 \times 21 \times 25$$

$$= 34650 \text{ cm}^3$$

(b)

$(k-3), (2k+1)$ and $(4k+3)$ are three consecutive terms of an A.P.

$\Rightarrow 2(2k+1) = (k-3) + (4k+3)$

$\Rightarrow 4k + 2 = k - 3 + 4k + 3$

$\Rightarrow 4k + 2 = 5k$

$\Rightarrow k = 2$

(c)

Given : PQRS is a cyclic quadrilateral.

$\angle QPS = 73°, \angle PQS = 55°$ and $\angle PSR = 82°$

(i) Opposite angle of a cyclic quadrilateral are supplementary.

$\Rightarrow \angle QPS + \angle QRS = 180°$

$\Rightarrow 73° + \angle QRS = 180°$

$\Rightarrow \angle QRS = 180° - 73° = 107°$

(ii) Opposite angle of a cyclic quadrilateral are supplementary.

$\Rightarrow \angle PSR + \angle PQR = 180°$

$\Rightarrow \angle PSR + (\angle PQS + \angle RQS) = 180°$

$\Rightarrow 82° + 55° + \angle RQS = 180°$

$\Rightarrow \angle RQS = 180° - 137° = 43°$

(iii) In ΔPQS, by angle sum property, we have

$\Rightarrow \angle PSQ + \angle PQS + \angle QPS = 180°$

$\Rightarrow \angle PSQ + 55° + 73° = 180°$

$\Rightarrow \angle PSQ = 180° - 128° = 52°$

Now, $\angle PRQ = \angle PSQ$ (angles in the same segment of a circle)

$\Rightarrow \angle PRQ = 52°$

3.

(a)

Given $(x+2)$ is a factor of x^3+ax+b;

$\Rightarrow (-2)^3+a(-2)+b=0 \qquad (x+2=0 \Rightarrow x=-2)$

$\Rightarrow -8-2a+b=0$

$\Rightarrow -2a+b=8 \quad ....(i)$

Also, given that $(x+3)$ is a factor of x^3+ax+b;

$\Rightarrow (-3)^3+a(-3)+b=0$

$\Rightarrow -27-3a+b=0$

$\Rightarrow -3a+b=27 \quad ....(ii)$

Subtracting (i) from (ii), we have

$-a=19 \Rightarrow a=-19$

Substituting $a=-19$ in (i), we have

$-2\times(-19)+b=8$

$\Rightarrow 38+b=8$

$\Rightarrow b=-30$

Hence, $a=-19$ and $b=-30$

(b)

$$\text{L.H.S.}=\sqrt{\sec^2\theta+\operatorname{cosec}^2\theta}$$

$$=\sqrt{\frac{1}{\cos^2\theta}+\frac{1}{\sin^2\theta}}$$

$$=\sqrt{\frac{\sin^2\theta+\cos^2\theta}{\cos^2\theta\sin^2\theta}}$$

$$=\sqrt{\frac{1}{\cos^2\theta\sin^2\theta}}$$

$$=\sqrt{\frac{1}{\cos^2\theta}\times\frac{1}{\sin^2\theta}}$$

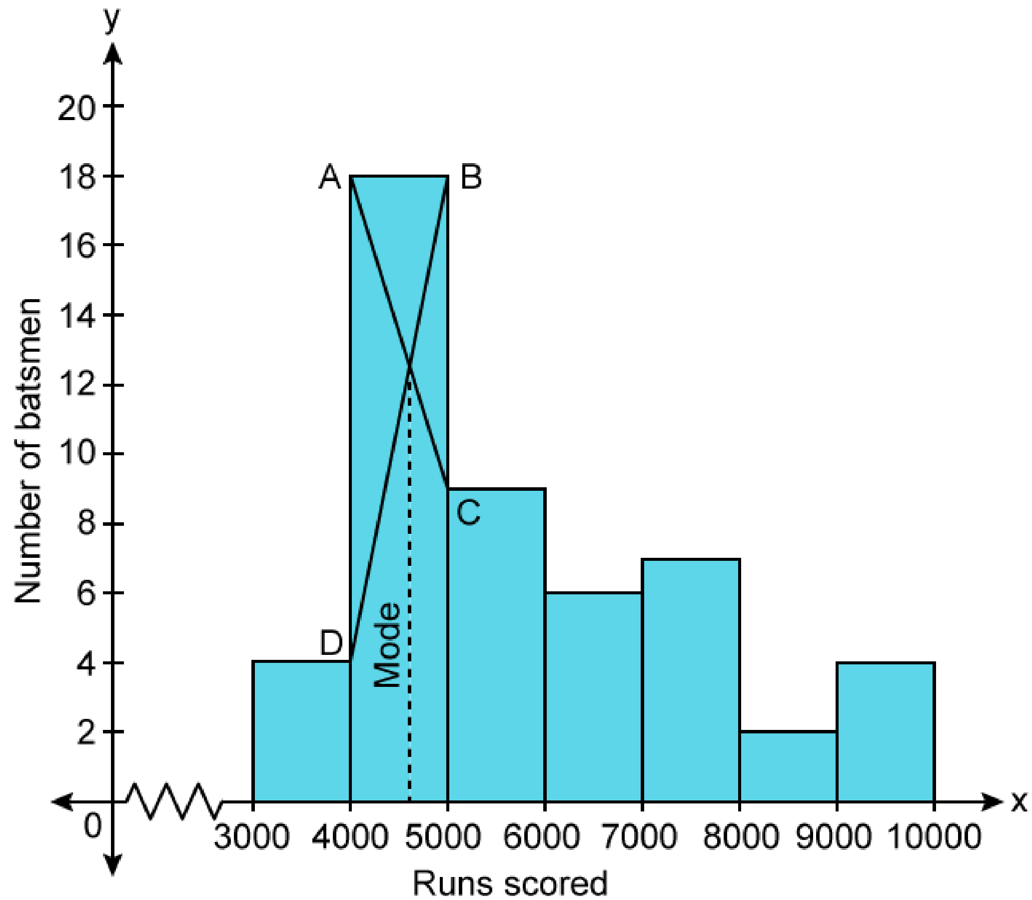
$$=\sqrt{\sec^2\theta\times\operatorname{cosec}^2\theta}$$

$$=\sec\theta\times\operatorname{cosec}\theta$$

$$\text{R.H.S.}=\tan\theta+\cot\theta=\frac{\sin\theta}{\cos\theta}+\frac{\cos\theta}{\sin\theta}=\frac{\sin^2\theta+\cos^2\theta}{\cos\theta\sin\theta}=\frac{1}{\cos\theta}\times\frac{1}{\sin\theta}=\sec\theta\times\operatorname{cosec}\theta$$

Thus, L.H.S. = R.H.S.

(c) The histogram is as follows:



From histogram, we have mode = 4600

4.

(a)

Given inequation: $-2+10x \le 13x+10 < 24+10x, \quad x \in Z$

$\Rightarrow -2+10x \le 13x+10 \quad$ and $\quad 13x+10 < 24+10x$

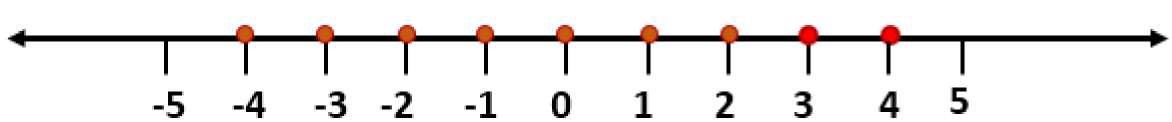
$\Rightarrow -2-10 \le 13x-10x \quad$ and $\quad 13x-10x < 24-10$

$\Rightarrow -12 \le 3x \quad$ and $\quad 3x < 14$

$\Rightarrow -4 \le x \quad$ and $\quad x < 4.6$

$\therefore$ Solution set $= \{x : -4 \le x < 4.6$ and $x \in Z)$

Representation on number line is as follows:



(b)

$$3x - 5y = 7$$
$$\Rightarrow 5y = 3x - 7$$
$$\Rightarrow y = \frac{3}{5}x - \frac{7}{5}$$
$$\Rightarrow \text{Its slope} = \frac{3}{5}$$
$$4x + ay + 9 = 0$$
$$\Rightarrow ay = -4x - 9$$
$$\Rightarrow y = \frac{-4}{a}x - \frac{9}{a}$$
$$\Rightarrow \text{Its slope} = \frac{-4}{a}$$

Since lines are perpendicular to each other,

$$\frac{3}{5} \times \frac{-4}{a} = -1 \Rightarrow \frac{3}{5} \times \frac{4}{a} = 1 \Rightarrow \frac{4}{a} = \frac{5}{3}$$
$$\Rightarrow a = \frac{4 \times 3}{5} = \frac{12}{5}$$

(c)

Given quadratic equation is $x^2 + 7x = 7$

$$\Rightarrow x^2 + 7x - 7 = 0$$

Comparing with $ax^2 + bx + c = 0$, we have $a = 1$, $b = 7$ and $c = -7$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$\Rightarrow x = \frac{-7 \pm \sqrt{7^2 - 4 \times 1 \times (-7)}}{2 \times 1}$$
$$\Rightarrow x = \frac{-7 \pm \sqrt{77}}{2}$$
$$\Rightarrow x = \frac{-7 \pm 8.77}{2}$$
$$\Rightarrow x = \frac{-7 + 8.77}{2} \text{ and } x = \frac{-7 - 8.77}{2}$$
$$\Rightarrow x = \frac{1.77}{2} \text{ and } x = \frac{-15.77}{2}$$
$$\Rightarrow x = 0.885 \text{ and } x = -7.885$$
$$\Rightarrow x = 0.89 \text{ and } x = -7.89 \text{ (correct to two decimal places)}$$

SECTION B (40 Marks)

Attempt any four questions from this section

5.

(a)

4^{th} term of G.P. $= 16$

$\Rightarrow ar^{4-1} = 16$

7^{th} term of G.P. $= 128$

$\Rightarrow ar^{7-1} = 128$

so, $\dfrac{ar^3}{ar^6} = \dfrac{16}{128}$

$\Rightarrow \dfrac{1}{r^3} = \dfrac{1}{8}$

$\Rightarrow r = 2$

$ar^3 = 16$

$a \times 2^3 = 16$

$a \times 8 = 16$

$a = 2$

(b)

Total investment $=$ Rs.22,500

Face value $=$ Rs.50

Discount $= \dfrac{10}{100} \times 50 =$ Rs.5

Market value $=$ Face value $-$ discount $=$ Rs.45

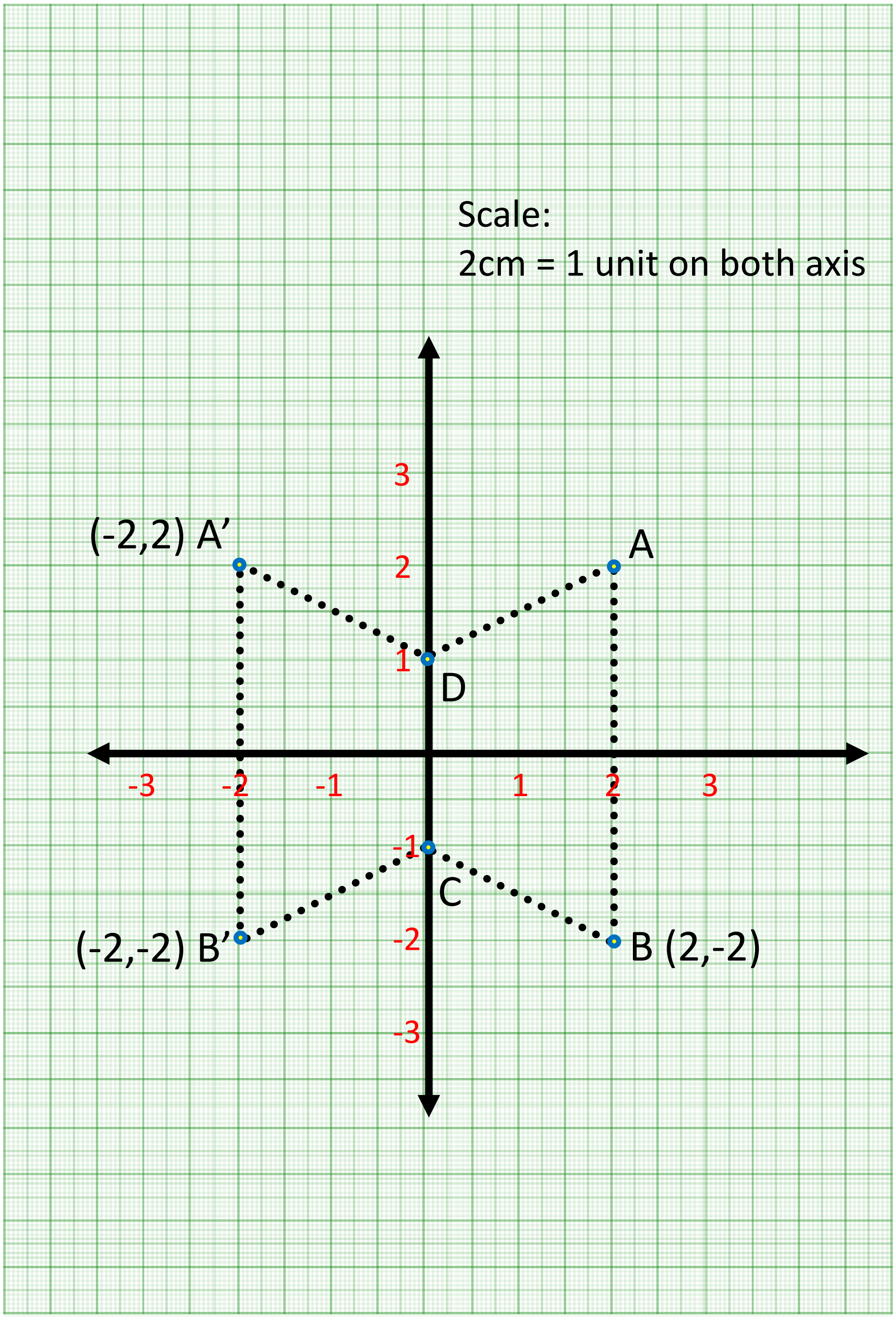
Total shares purchased $= \dfrac{22{,}500}{45} = 500$

Total dividend $= \dfrac{12}{100} \times 50 \times 500 = 3000$

Rate of return $= \dfrac{3000}{22500} \times 100 = 13.33\% \approx 13\%$

(c)

(i) and (ii)



(iii) D and C are invariant points.
(iv) A'B'CD is a trapezium

6.

(a)

$$\frac{2x+\sqrt{4x^2-1}}{2x-\sqrt{4x^2-1}}=4$$

$$\Rightarrow \frac{2x+\sqrt{4x^2-1}+2x-\sqrt{4x^2-1}}{2x+\sqrt{4x^2-1}-2x+\sqrt{4x^2-1}}=\frac{4+1}{4-1} \quad \text{(By componendo} - \text{dividendo)}$$

$$\Rightarrow \frac{4x}{2\sqrt{4x^2-1}}=\frac{5}{3}$$

$$\Rightarrow \frac{2x}{\sqrt{4x^2-1}}=\frac{5}{3}$$

$$\Rightarrow \frac{4x^2}{4x^2-1}=\frac{25}{9} \quad \text{(squaring both sides)}$$

$$\Rightarrow \frac{4x^2-4x^2+1}{4x^2-1}=\frac{25-9}{9} \quad \text{(By dividendo)}$$

$$\Rightarrow \frac{1}{4x^2-1}=\frac{16}{9}$$

$$\Rightarrow 9=64x^2-16$$

$$\Rightarrow 64x^2=25$$

$$\Rightarrow x^2=\frac{25}{64}$$

$$\Rightarrow x=\pm\frac{5}{8}$$

$$\Rightarrow x=\frac{5}{8} \quad \text{(x is positive)}$$

(b)

$$\text{Given}: A=\begin{bmatrix}2 & 3\\ 5 & 7\end{bmatrix},\ B=\begin{bmatrix}0 & 4\\ -1 & 7\end{bmatrix} \text{ and } C=\begin{bmatrix}1 & 0\\ -1 & 4\end{bmatrix}$$

$$\text{Now, } AC = A\times C=\begin{bmatrix}2 & 3\\ 5 & 7\end{bmatrix}\times\begin{bmatrix}1 & 0\\ -1 & 4\end{bmatrix}$$

$$=\begin{bmatrix}2\times1+3\times(-1) & 2\times0+3\times4\\ 5\times1+7\times(-1) & 5\times0+7\times4\end{bmatrix}$$

$$=\begin{bmatrix}2-3 & 0+12\\ 5-7 & 0+28\end{bmatrix}$$

$$=\begin{bmatrix}-1 & 12\\ -2 & 28\end{bmatrix}$$

$$\text{And, } B^2 = B\times B=\begin{bmatrix}0 & 4\\ -1 & 7\end{bmatrix}\times\begin{bmatrix}0 & 4\\ -1 & 7\end{bmatrix}$$

$$=\begin{bmatrix}0\times0+4\times(-1) & 0\times4+4\times7\\ -1\times0+7\times(-1) & -1\times4+7\times7\end{bmatrix}$$

$$=\begin{bmatrix}0-4 & 0+28\\ 0-7 & -4+49\end{bmatrix}$$

$$=\begin{bmatrix}-4 & 28\\ -7 & 45\end{bmatrix}$$

$$\text{Now, } AC+B^2-10C=\begin{bmatrix}-1 & 12\\ -2 & 28\end{bmatrix}+\begin{bmatrix}-4 & 28\\ -7 & 45\end{bmatrix}-10\begin{bmatrix}1 & 0\\ -1 & 4\end{bmatrix}$$

$$=\begin{bmatrix}-1 & 12\\ -2 & 28\end{bmatrix}+\begin{bmatrix}-4 & 28\\ -7 & 45\end{bmatrix}-\begin{bmatrix}10 & 0\\ -10 & 40\end{bmatrix}$$

$$=\begin{bmatrix}-1-4-10 & 12+28-0\\ -2-7+10 & 28+45-40\end{bmatrix}$$

$$=\begin{bmatrix}-15 & 40\\ 1 & 33\end{bmatrix}$$

(c)

$$\begin{aligned}
\text{L.H.S.} &= (1+\cot\theta-\operatorname{cosec}\theta)(1+\tan\theta+\sec\theta)\\
&= \left(1+\frac{\cos\theta}{\sin\theta}-\frac{1}{\sin\theta}\right)\left(1+\frac{\sin\theta}{\cos\theta}+\frac{1}{\cos\theta}\right)\\
&= \left(\frac{\sin\theta+\cos\theta-1}{\sin\theta}\right)\left(\frac{\cos\theta+\sin\theta+1}{\cos\theta}\right)\\
&= \frac{1}{\sin\theta\cos\theta}\begin{pmatrix}\sin\theta\cos\theta+\sin^2\theta+\sin\theta+\cos^2\theta\\ +\sin\theta\cos\theta+\cos\theta-\cos\theta-\sin\theta-1\end{pmatrix}\\
&= \frac{1}{\sin\theta\cos\theta}\left(2\sin\theta\cos\theta+(\sin^2\theta+\cos^2\theta)-1\right)\\
&= \frac{1}{\sin\theta\cos\theta}\left(2\sin\theta\cos\theta+1-1\right)\\
&= \frac{2\sin\theta\cos\theta}{\sin\theta\cos\theta}\\
&= 2\\
&= \text{R.H.S.}
\end{aligned}$$

7.

(a)

For the given equation $x^2+4kx+(k^2-k+2)=0$

$a=1,\ b=4k$ and $c=k^2-k+2$

Since the roots are equal,

$b^2-4ac=0$

$\Rightarrow (4k)^2-4\times1\times(k^2-k+2)=0$

$\Rightarrow 16k^2-4k^2+4k-8=0$

$\Rightarrow 12k^2+4k-8=0$

$\Rightarrow 3k^2+k-2=0$

$\Rightarrow 3k^2+3k-2k-2=0$

$\Rightarrow 3k(k+1)-2(k+1)=0$

$\Rightarrow (k+1)(3k-2)=0$

$\Rightarrow k+1=0$ or $3k-2=0$

$\Rightarrow k=-1$ or $k=\dfrac{2}{3}$

(b)

Scale: $1:50000$

1 cm represents 50000 cm $= \dfrac{50000}{1000 \times 100} = 0.5$ km

(i) In ΔABC, by pythagoras theorem

$$AC^2 = AB^2 + BC^2 = 6^2 + 8^2 = 36 + 64 = 100$$

$\Rightarrow AC = 10$ cm

$\Rightarrow$ Actual length of diagonal $AC = 10 \times 0.5 = 5$ km

(ii) 1 cm $= 0.5$ km

$\Rightarrow 1\ \text{cm}^2 = 0.25$ km

Area of rectangle ABCD $= AB \times BC = 6 \times 8 = 48\ \text{cm}^2$

$\Rightarrow$ Actual area of a plot $= 48 \times 0.25 = 12\ \text{km}^2$

(c)

Let the co-ordinates of M be (x, y).

Thus, we have

$$x = \frac{m_1x_2 + m_2x_1}{m_1 + m_2} = \frac{1\times(-1) + 2\times 2}{1+2} = \frac{-1+4}{3} = \frac{3}{3} = 1$$

$$y = \frac{m_1y_2 + m_2y_1}{m_1 + m_2} = \frac{1\times(2) + 2\times 5}{1+2} = \frac{2+10}{3} = \frac{12}{3} = 4$$

$\Rightarrow$Co-ordinates of M are $(1,4)$.

Slope of line passing through C and M $= m = \dfrac{4-8}{1-5} = \dfrac{-4}{-4} = 1$

$\therefore$ Required equation is given by

$$y - 8 = 1(x - 5)$$

$$\Rightarrow y - 8 = x - 5$$

$$\Rightarrow y = x + 3$$

8.

(a)

Let the original number of children be x.

It is given that Rs. 7500 is divided among x children.

$\Rightarrow$ Money received by each child $=$ Rs.$\dfrac{7500}{x-20}$

If there were 20 less children, then money received by each child $=$ Rs. $\dfrac{7500}{x-20}$

From the given information, we have

$$\frac{7500}{x-20}-\frac{7500}{x}=100$$

$$\Rightarrow \frac{75}{x-20}-\frac{75}{x}=1$$

$$\Rightarrow \frac{75x-75x+1500}{x^2-20x}=1$$

$$\Rightarrow 1500=x^2-20x$$

$$\Rightarrow x^2-20x-1500=0$$

$$\Rightarrow x^2-50x+30x-1500=0$$

$$\Rightarrow x(x-50)+30(x-50)=0$$

$$\Rightarrow (x-50)(x+30)=0$$

$$\Rightarrow x=50 \text{ or } x=-30$$

Since number of children cannot be negative, we reject $x=-30$.

$$\Rightarrow x=50$$

Thus, the original number of children $=50$

(b)

We have,

C.I.	f	Class mark x	fx
$0-10$	7	5	35
$10-20$	a	15	15a
$20-30$	8	25	200
$30-40$	10	35	350
$40-50$	5	45	225
	$\sum f=30+a$		$\sum fx=810+15a$

Mean $=24$ (given)

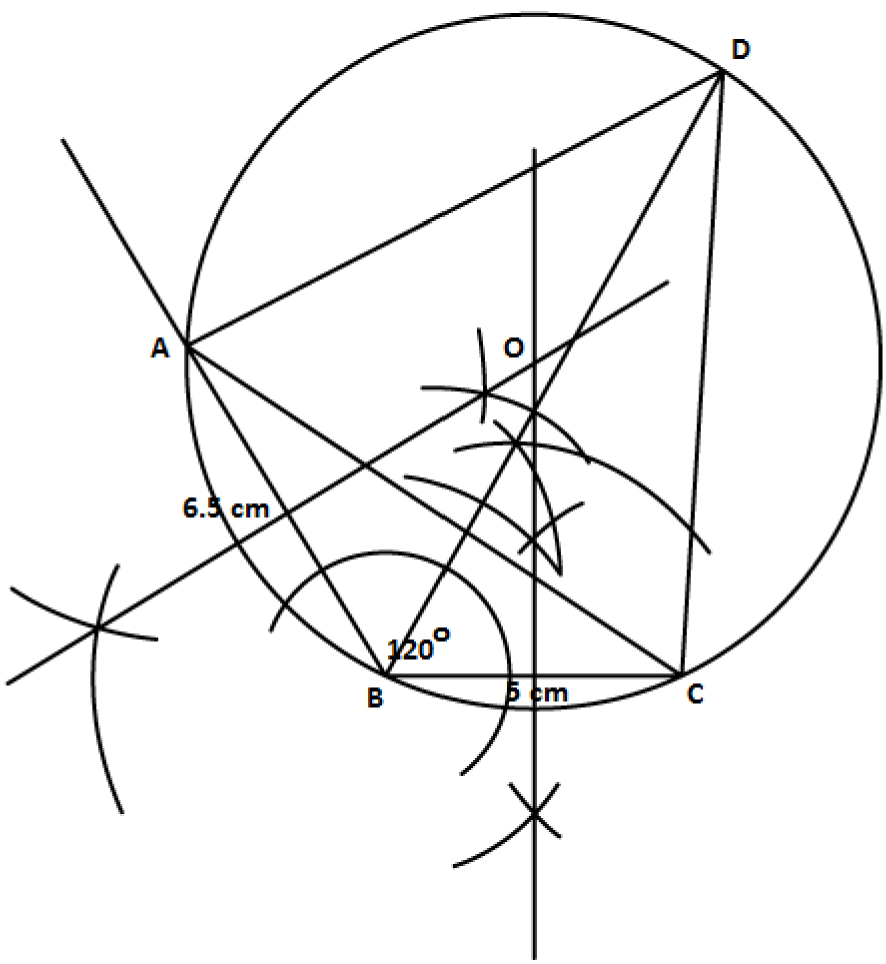
$$\Rightarrow \frac{\sum fx}{\sum f}=24 \Rightarrow \frac{810+15a}{30+a}=24 \Rightarrow 810+15a=720+24a$$

$$\Rightarrow a=10$$

(c)

Steps of construction:

1) Draw a line segment BC of length 5 cm.
2) At B, draw a ray BX making an angle of 120° with BC.
3) With B as centre and radius 6.5 cm, draw an arc to cut the ray BX at A. Join AC.
   ΔABC will be obtained.
4) Draw the perpendicular bisectors of AB and BC to meet at point O.
5) With O as centre and radius OA, draw a circle. The circle will circumscribe ΔABC.
6) Draw the angle bisector of ∠ABC.
7) The angle bisector of ∠ABC and let it meet circle at point D.
8) Join AD and DC to obtain the required cyclic quadrilateral ABCD such that point D is equidistant from AB and BC.



9.

(a)

Given : $P = \text{Rs. } 1000$, $r = 10\%$ and $I = \text{Rs. } 5550$

$$I = P \times \frac{n(n+1)}{2 \times 12} \times \frac{r}{100}$$

$$\Rightarrow 5550 = 1000 \times \frac{n(n+1)}{24} \times \frac{10}{100}$$

$$\Rightarrow 1332 = n(n+1)$$

$$\Rightarrow n^2 + n - 1332 = 0$$

$$\Rightarrow n^2 + 37n - 36n - 1332 = 0$$

$$\Rightarrow n(n+37) - 36(n+37) = 0$$

$$\Rightarrow (n+37)(n-36) = 0$$

$$\Rightarrow n = -37 \text{ or } n = 36$$

Since number of months cannot be negative, we reject $n = -37$

$$\Rightarrow n = 36$$

Thus, total time is 36 months.

(b)

(i) In ΔPMN and ΔPQR, $MN \parallel QR$

$\Rightarrow \angle PMN = \angle PQR$ (alternate angles)

$\Rightarrow \angle PNM = \angle PRQ$ (alternate angles)

$\Rightarrow \Delta PMN \sim \Delta PQR$ (AA postulate)

$$\Rightarrow \frac{PM}{PQ} = \frac{MN}{QR}$$

$$\Rightarrow \frac{2}{5} = \frac{MN}{QR} \quad \left[\frac{PM}{MQ} = \frac{2}{3} \Rightarrow \frac{PM}{PQ} = \frac{2}{5}\right]$$

(ii) In ΔOMN and ΔORQ,

$\angle OMN = \angle ORQ$ (alternate angles)

$\angle MNO = \angle OQR$ (alternate angles)

$\Rightarrow \Delta OMN \sim \Delta ORQ$ (AA postulate)

(iii) $$\frac{\text{Area of } \Delta OMN}{\text{Area of } \Delta ORQ} = \frac{MN}{RQ} = \frac{2}{5}$$

(c)

$$\text{Volume of solid} = \text{Volume of cone} + \text{Volume of cylinder}$$
$$+ \text{Volume of hemisphere}$$

$$\text{Volume of cone} = \frac{\pi r^2 h}{3} = \frac{22 \times 7 \times 7 \times 4}{7 \times 3} = \frac{616}{3}\ \text{cm}^3$$

$$\text{Volume of cylinder} = \pi r^2 h = \frac{22 \times 7 \times 7 \times 4}{7} = 616\ \text{cm}^3$$

$$\text{Volume of hemisphere} = \frac{2}{3}\pi r^3 = \frac{2 \times 22 \times 7 \times 7 \times 7}{3 \times 7} = \frac{2156}{3}\ \text{cm}^3$$

$$\text{Total volume} = \frac{616}{3} + 616 + \frac{2156}{3} = 1540\ \text{cm}^3$$

10.

(a) Let P(x) = $2x^3 + 3x^2 - 9x - 10$
P(2) = 16 + 12 – 18 – 10
P(2) = 0

So, (x – 2) is a factor.

Let us divide P(x) with (x-2), we get

$(x - 2)\ (2x^2 + 7x + 5)$

This can be further factored to

$(x - 2)\ (2x^2 + 5x + 2x + 5)$ ……… (Split 7x into two terms, whose sum is 7x and product is$10x^2$)

$(x - 2)\ (2x^2 + 5x + 2x + 5)$

$(x - 2)\ (x(2x + 5) + 1(2x + 5))$

$(x - 2)(2x + 5)(x + 1)$

(b)

Now,

OP = QR....given

So, OP = OT = OQ = QR

In ΔRQP

RQ = QO

So $\angle QRO = \angle QOR = 20^{o}$

So by sum of angles in ΔRQP

$\angle RQO = 140^{o}$

Now

$\angle RQO + \angle OQP = 180°$.....linear pair

$\angle OQP = 40^{o}$

In ΔPOQ

OQ = PO...radii

So $\angle QPO = \angle OQP = 40^{o}$

So by sum of angles in ΔOQP

$\angle POQ = 100^{o}$

Now,

$\angle POT + \angle POQ + \angle QOR = 180°$......angles in straight line

$x = 60^{o}$

(c)

In ΔPQR

$\tan 60° = \frac{RQ}{PQ}$

$\sqrt{3} = \frac{50}{PQ}$

$PQ = \frac{50}{\sqrt{3}}$

In ΔPQT

$\tan 30° = \frac{PT}{PQ}$

$\frac{1}{\sqrt{3}} = \frac{PT}{\frac{50}{\sqrt{3}}}$

$PT = \frac{1}{\sqrt{3}} \times \frac{50}{\sqrt{3}} = \frac{50}{3}$

11.

(a)

Let a be the first term and d be the common difference of given A.P.

Now,

4^{th} term $= 22$

$\Rightarrow a + 3d = 22 \quad ....(i)$

15^{th} term $= 66$

$\Rightarrow a + 14d = 66 \quad ....(ii)$

Subtracting (i) from (ii), we have

$11d = 44$

$\Rightarrow d = 4$

Substituting the value of d in (i), we get

$a = 22 - 3 \times 4 = 22 - 12 = 10$

$\Rightarrow$ First term $= 10$

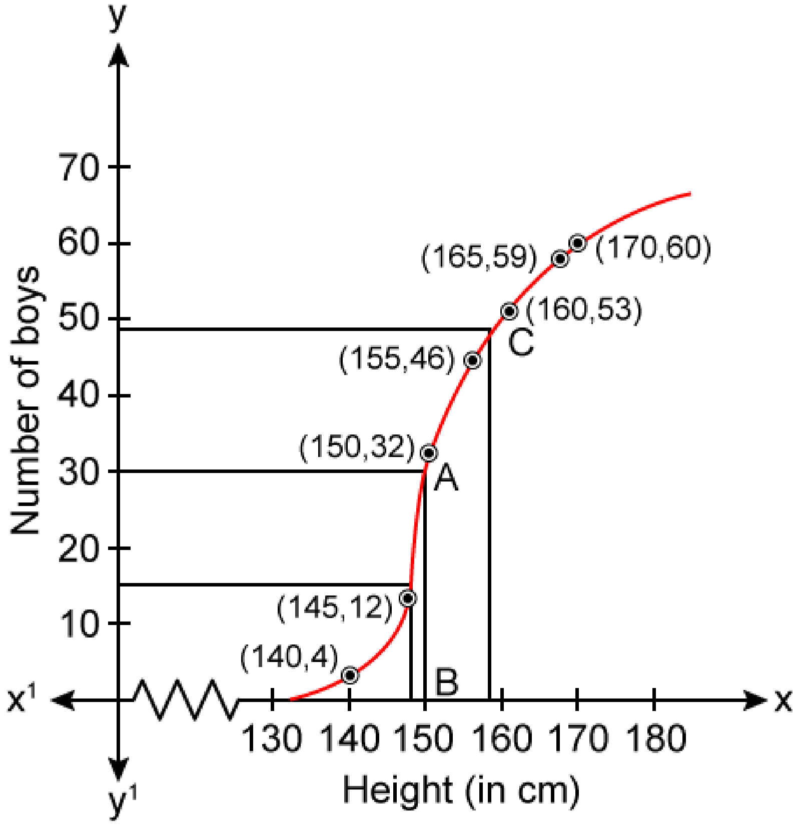
Now,

$$\text{Sum of 8 terms} = \frac{8}{2}[2 \times 10 + 7 \times 4] = 4[20 + 28] = 4 \times 48 = 192$$

(b) The cumulative frequency table of the given distribution table is as follows:

Height in cm	No. of boys (f)	Cumulative frequency
135-140	4	4
140-145	8	12
145-150	20	32
150-155	14	46
155-160	7	53
160-165	6	59
165-170	1	60

Plot the points (140, 4), (145, 12), (150, 32), (155, 46), (160, 53), (165, 59) and (170, 60) on a graph paper and join them to get an ogive.



Number of boys = N = 60

(i) Median $= \left(\frac{N}{2}\right)^{th}$ term $= \left(\frac{60}{2}\right)^{th}$ term $= 30^{th}$ term

Through mark 30 on the Y-axis, draw a horizontal line which meets the curve at point A.
Through point A, on the curve draw a vertical line which meets the X-axis at point B.
The value of point B on the X-axis is the median, which is 152.

(ii) Lower quartile $(Q_1) = \left(\frac{N}{4}\right)^{th}$ term $= \left(\frac{60}{4}\right)^{th}$ term $= 15^{th}$ term $= 148$

(iii) Through mark of 158 on X-axis, draw a verticle line which meets the graph at point C.
Then through point C, draw a horizontal line which meets the Y-axis at the mark of 48.
Thus, number of boys in the class who are tall $= 60 - 48 = 12$

ICSE Board

Class X

Mathematics

Board Paper 2019

(Two hours and a half)

Answers to this Paper must be written on the paper provided separately. You will not be allowed to write during the first 15 minutes.

This time is to be spent in reading the question paper.

The time given at the head of this Paper is the time allowed for writing the answers.

Attempt **all** questions from **Section A** and **any four questions** from **Section B.**

All working, including rough work, must be clearly shown and must be done on the same sheet as the rest of the answer.

Omission of essential working will result in loss of marks.

The intended marks for questions or parts of questions are given in brackets [ ].

Mathematical tables are provided.

SECTION A (40 Marks)

Attempt **all** questions from this Section.

Question 1

(a) Solve the following in equation and write down the solution set: [3]

$11x - 4 < 15x + 4 \leq 13x + 14,\ x \in W$

Represent the solution on a real number line.

Ans. The given inequality is:

$11x - 4 < 15x + 4 \leq 13x + 14$

which forms two cases that are:

Case 1: $15x+4>11x-4$

Case2: $15x+4\le 13x+14$

Solving the case 1, we have

$15x+4>11x-4$

$\Rightarrow 15x-11x>-4-4$
$\Rightarrow 4x>-8$
$\Rightarrow x>\frac{-8}{4}$
$\therefore x>-2$

Solving the case 2, we get

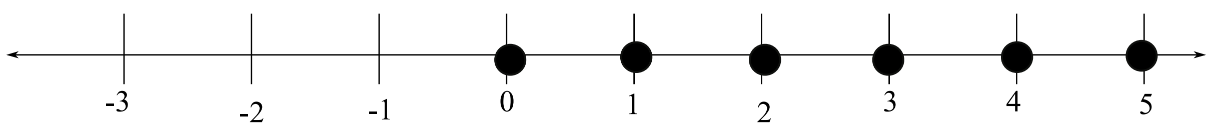
$15x+4\le 13x+14$

$\Rightarrow 15x-13x\le 14-4$
$\Rightarrow 2x\le 10$
$\Rightarrow x\le \frac{10}{2}$
$\therefore x\le 5$

So, we get $x>-2$ and $x\le 5$

$\Rightarrow -2<x\le 5$

Since $x\in w$, Therefore, x= {0, 1, 2, 3, 4, 5}



(b) A man invests Rs. 4500 in shares of a company which is paying 7.5% dividend. If Rs. 100 shares are available at a discount of 10%. Find: [3]

(i) Number of shares he purchases.

(ii) His annual income.

Ans. Given,

Total amount of investment = Rs. 4500

Dividend (%) = 7.5%

Face value = Rs. 100

Discount offered (%) = 10%

Market Value of share = 100 –10% 0f 100

= 100 – 10

= Rs.90

We know that, Numbers of share = $\frac{\text{Investment}}{\text{Market value}}$

$= \frac{4500}{90} = 50\,\text{shares}$

Also, Annual Income = $\frac{\text{Number of share} \times \text{Dividend percent} \times \text{Face value}}{100}$

$= \frac{50 \times 7.5 \times 100}{100}$

= Rs. 375.0

Hence, the total number of shares purchased is 50 and the annual income is Rs. 375

(c) In class of 40 students, marks obtained by the students a class test (out of 10) are given below,

Marks	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
Number of students	**1**	**2**	**3**	**3**	**6**	**10**	**5**	**4**	**3**	**3**

Calculate the following for the given distribution: [4]

(i) Median

(ii) Mode

Ans. Let the marks obtained be represented by x and the number of students be represented by f

Then, we can arrange the data as:

Marks(x)	Number of students(f)	(fx)
1	1	1
2	2	4
3	3	9
4	3	12
5	6	30
6	10	60
7	5	35
8	4	32
9	3	27
10	3	30
Total	$\sum f = 40$	$\sum fx = 240$

i. We know, Mean = $\dfrac{\sum fx}{\sum f}$

$$= \frac{240}{40} = 6$$

ii. Mode = Class with highest frequency

Here in the above table, we see 10 is the highest frequency

Hence, Mode =6

Question 2

(a) Using the factor theorem, show that (x - 2) is a factor of $x^3 + x^2 - 4x - 4$ **, Hence, factorise the polynomial completely. [3]**

Ans. Let's take the given polynomial be P(x).

We have P(x) = $x^3 + x^2 - 4x - 4$

According to question, (x - 2) is a factor of given polynomial

We know that, under factor theorem, x-a is a factor of P(x) only if P(a) = 0

So, to prove that, we substitute the value of x by 2

$$P(2) = 2^3 + 2^2 - 4\times 2 - 4$$
$$P(2) = 8 + 4 - 8 - 4$$
$$P(2) = 0$$

Therefore, x-2 is a factor of P(x)

Now, factorising the polynomial, we get

$$x^3 + x^2 - 4x - 4$$
$$= x^3 - 4x + x^2 - 4$$
$$= x(x^2 - 4) + 1(x^2 - 4) \qquad [\because (a^2 - b^2) = (a+b)(a-b)]$$
$$= (x^2 - 4)(x + 1)$$
$$= (x + 2)(x - 2)(x + 1)$$

(b) Prove that: (cosec θ - sin θ)(sec θ – cos θ) (tan θ + cot θ) = 1 [3]

Ans. Given, L.H.S= (cosec θ - sin θ)(sec θ – cos θ) (tan θ + cot θ)

$$\Rightarrow \left(\frac{1}{\text{Sin}\theta}\text{-Sin}\theta\right)\left(\frac{1}{\text{Cos}\theta}\text{-Cos}\theta\right)\left(\frac{\text{Sin}\theta}{\text{Cos}\theta}+\frac{\text{Cos}\theta}{\text{Sin}\theta}\right)$$

$$=\left(\frac{1\text{-Sin}^2\theta}{\text{Sin}\theta}\right)\left(\frac{1\text{-Cos}^2\theta}{\text{Cos}\theta}\right)\left(\frac{\text{Sin}^2\theta\text{+Cos}^2\theta}{\text{Sin}\theta.\text{Cos}\theta}\right)$$

$$=\frac{\text{Cos}^2\theta}{\text{Sin}\theta}.\frac{\text{Sin}^2\theta}{\text{Cos}\theta}.\frac{1}{\text{Sin}\theta.\text{Cos}\theta}$$

$=1$

$=$R.H.S

Hence, L.H.S = R.H. S proved.

(c) In an Arithmetic Progression (A.P.) the fourth and sixth terms are 8 and 14 respectively, Find the:[4]

(i) First term

(ii) Common difference

(iii) Sum of the first 20 terms

Ans.

Let the first term of A.P be 'a' and common difference be 'd'.

Then using formula $a_n = a+(n-1)d$ to get n^{th} term, we get

$a_4 = a+(4-1)d$

$\Rightarrow 8=a+3d$................(i)

[Given: Fourth term is 8 and sixth term is 14]

Also,

$a_6 = a+(6-1)d$

$\Rightarrow 14=a+5d$..............(ii)

Now, solving (i) and (ii) simultaneously, we get

a+3d-a-5d = 8-14

$\Rightarrow$ -2d =-6

$\therefore$ d = 3

Putting value of d in (i), we get

a +3×3 = 8

$\Rightarrow$a+9 = 8

$\therefore$ a = -1

We know that, the sum of first n terms = $\frac{n}{2}\left[2a+(n-1)d\right]$

So, the sum of first 20 terms $S_{20} = \frac{20}{2}\left[2(-1)+(20-1)3\right]$

$= 10(-2+19\times 3)$

$= 10(-2+57)$

$= 10\times 55$

$= 550$

Question 3

(a) Simplify: [3]

$$\text{SinA}\begin{bmatrix} \text{SinA} & \text{-CosA} \\ \text{CosA} & \text{SinA} \end{bmatrix} + \text{CosA}\begin{bmatrix} \text{CosA} & \text{SinA} \\ \text{-SinA} & \text{CosA} \end{bmatrix}$$

Ans.

$$\text{SinA}\begin{bmatrix}\text{SinA} & \text{-CosA}\\ \text{CosA} & \text{SinA}\end{bmatrix}+\text{CosA}\begin{bmatrix}\text{CosA} & \text{SinA}\\ \text{-SinA} & \text{CosA}\end{bmatrix}$$

$$=\begin{bmatrix}\text{Sin}^2\text{A} & \text{-SinA.CosA}\\ \text{SinA.CosA} & \text{Sin}^2\text{A}\end{bmatrix}+\begin{bmatrix}\text{Cos}^2\text{A} & \text{CosA.SinA}\\ \text{-CosA.SinA} & \text{Cos}^2\text{A}\end{bmatrix}$$

$$=\begin{bmatrix}\text{Sin}^2\text{A+Cos}^2\text{A} & \text{-SinA.CosA+CosA.SinA}\\ \text{SinA.CosA+(-CosA.SinA)} & \text{Sin}^2\text{A+Cos}^2\text{A}\end{bmatrix}$$

$$=\begin{bmatrix}1 & \text{-SinA.CosA+CosA.SinA}\\ \text{SinA.CosA-CosA.SinA)} & 1\end{bmatrix}[\because \text{Sin}^2\text{A+Cos}^2\text{A=1}]$$

$$=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$$

(b) M and N are two points on the X axis and Y axis respectively. P(3, 2) divides the line segment MN in the ratio 2 : 3. Find: [3]

(i) The coordinates of M and N

(ii) Slope of the line MN.

Ans.

Let the co ordinates of M and N be (a,0) and (0,b) on x-axis and y- axis respectively

We know that,

The co ordinates of the point which divides the line segment joining $(x_1, y_1) and (x_2, y_2)$ internally in the ratio m:n is $\left(\frac{mx_2+nx_1}{m+n}, \frac{my_2+ny_1}{m+n}\right)$

According to question, P(3,2) divides the line joining M and N internally in the ratio 2:3,

Thus, using the above formula, we get

$\left(\frac{3a}{5}, \frac{2b}{5}\right)$=(3,2)

So,

$\frac{3a}{5}$=3

$\Rightarrow$ a=$\frac{3\times5}{3}$

$\therefore$ a=5

And

$\frac{2b}{5}$=2

$\Rightarrow$ b=$\frac{2\times5}{2}$

$\therefore$ b=5

Therefore, the required coordinates of M and N are (5,0) and (0,5)

Now,

M(5,0) and N(0,5)

We know, slope of the line = $\frac{y_2 - y_1}{x_2 - x_1}$

Therefore, Slope of line MN = $\frac{5-0}{0-5} = \frac{5}{-5} = -1$

(c) A solid metallic sphere of radius 6 cm is melted and made into a solid cylinder of height 32cm. Find the: [4]

(i) Radius of the cylinder

(ii) Curved surface area of the cylinder

Take π = 3.1

Ans.

(i) Given, radius of metallic sphere(r) = 6cm

We know, volume of sphere = $\frac{4}{3}\pi r^3$

$\therefore$ Volume of metallic sphere = $\frac{4}{3}\pi(6)^3$

Let r_1 be radius of the cylinder

Height of the cylinder (h) = 32cm

We know, volume of cylinder = $\pi r^2 h$

$\therefore$ Volume of given cylinder = $\pi r_1^2 \times 32$

Now, according to question,

Volume of sphere = Volume of cylinder

$$\Rightarrow \frac{4}{3}\pi(6)^3 = \pi r_1^2 \times 32$$
$$\Rightarrow \frac{4}{3}\times 216 = 32 r_1^2$$
$$\Rightarrow \frac{4\times 216}{3\times 32} = r_1^2$$
$$\Rightarrow r_1^2 = 9$$
$$\therefore r_1 = 3$$

Hence, the radius of the cylinder is 3cm

(ii.) Height of cylinder (h) = 32 cm and radius(r) = 3cm

So, The curved surface area of cylinder = $2\pi rh$

$= 2\times 3.1\times 3\times 32$

= 595.2

Question 4

(a) The following numbers, K + 3, K + 2, 3K - 7 and 2K – 3 are in proportion. Find K. [3]

Ans. Given that, K + 3, K + 2, 3K - 7 and 2K – 3 are in proportion.

$\therefore \frac{\text{K+3}}{\text{K+2}} = \frac{\text{3K}-7}{2K-3}$

$\Rightarrow$ (K+3)(2K-3) = (3K-7)(K+2)

$\Rightarrow$ $2K^2$-3K+6K-9 = $3K^2$+6K-7K-14

$\Rightarrow$ $2K^2$+3K-9=$3K^2$-K-14

$\Rightarrow$ K^2-4K-5=0

$\Rightarrow$ K^2-5K+K-5=0

$\Rightarrow$ (K-5)(K+1) = 0

$\therefore \text{K} = 5 or -1$

(b) Solve for x the quadratic equation $x^2 - 4x - 8 = 0$. Give your answer correct to three significant figures. [3]

Ans.

Given equation, $x^2 - 4x - 8 = 0$

Comparing it with the quadratic equation $ax^2 + bx + c = 0$, we get

a= 1, b = -4 and c = -8

$\therefore b^2 - 4ac = (-4)^2 - 4 \times 1 \times (-8)$

= 16+32

= 48

Substituting this value in the formula x = $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

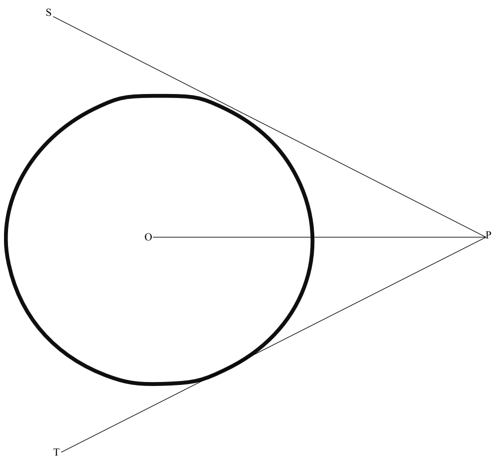
We get,

$$x = \frac{-(-4) \pm \sqrt{48}}{2 \times 1}$$
$$\Rightarrow x = \frac{4 \pm 4\sqrt{3}}{2}$$
$$\Rightarrow x = 2 \pm 2\sqrt{3}$$
$$\Rightarrow x = 2 \pm 2 \times 1.732$$
$$\Rightarrow x = 2 \pm 3.464$$
$$\therefore x = 5.464 \text{ or } -1.464$$

(c) Use ruler and compass only for answering this question. Draw a circle of radius 4 cm. Mark the centre as O. Mark a point P outside the circle at a distance of 7 cm from the centre, Construct two tangents to the circle from the external point P. Measure and write down the length of any one tangent.

Ans. Steps for construction are as below:

i. Take measure 4 cm in compass and draw a circle, with center as O.
ii. Draw a straight line from O to P, such that OP = 7cm
iii. Now find the midpoint of OP by drawing a perpendicular bisector
iv. Mark the midpoint as X
v. Take measure of XO in the compass and cut arcs at S and T on the Circle
vi. Join PS and PT
vii. Measure of PS comes out to be 5.74 cm



SECTION B (40 Marks)

Attempt any **four** questions from this Section.

Question 5

(a) There are 25 discs numbered 1 to 25. They are put in a closed box and shaken thoroughly. A disc is drawn at random from the box. [3]

Find the probability that the number on the disc is:

(i) An odd number

(ii) Divisible by 2 and 3 both.

(iii) A number less than 16.

Ans.

According to question,

Total number of discs numbered 1 to 25 = 25

So, the number of possible outcomes in the sample space is n(S) = 25

We know , Probability of an event $= \dfrac{\text{Number of favourable outcomes}}{\text{Total number of possible outcomes}}$

i. Let A be the event of getting an odd number

$\therefore$ A = {1, 3, 5, 7, 9, 11, 13, 15, 17, 19, 21, 23, 25}

$\therefore$ n(A) = 13

Therefore, the probability of getting an odd number = $\frac{n(A)}{n(S)} = \frac{13}{25}$

ii. Let B be the event of getting a number divisible by 2 and 3. To get the numbers which are divisible by 2 and 3 both, we need to find the numbers which are divisible by 6.

$\therefore$ B = {6, 12, 18, 24}

$\therefore$ n(B) = 4

Therefore, the probability of getting an odd number = $\frac{n(B)}{n(S)} = \frac{4}{25}$

iii. Let C be the event of getting a number less than 16.

$\therefore$ C = {1, 2, 3, 4, 5, 6,7 , 8, 9, 10, 11, 12, 13, 14,15}

$\therefore$ n(C) = 15

Therefore, the probability of getting a number less than 16 = $\frac{n(C)}{n(S)} = \frac{15}{25} = \frac{3}{5}$

(b) Rekha opened a recurring deposit account for 20 months. The rate of interest is 9% per annum and Rekha receives Rs. 441 as interest at the time of maturity. Find the amount Rekha deposited each month. [3]

Ans.

Given, Rate of interest given by bank (r) = 9%

Time period (n) = 20 months

Interest at the time of maturity = Rs.441

Let, the principle deposited every month be P

According to question,

$$P\frac{n(n+1)}{2}\times\frac{r}{12\times100}=441$$
$$\Rightarrow P\frac{20(20+1)}{2}\times\frac{9}{1200}=441$$
$$\Rightarrow P\frac{20\times21}{2}\times\frac{9}{1200}=441$$
$$\Rightarrow P\times1.575=441$$
$$\therefore P=280$$

Therefore, the amount deposited by Rekha each month is Rs 280.

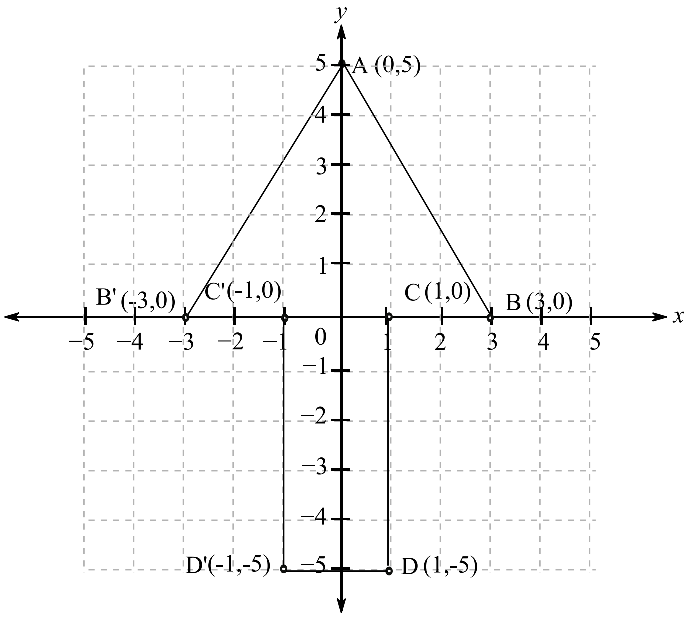
(c) Use a graph sheet for this question. [4]
Take 1 cm = 1 unit along both x and y axis.

(i) Plot the following points:
A(0,5), B(3,0), C(1,0), and D(1,-5)

(ii) Reflect the points B, C, and D on the y axis and name them as B', C' and D' respectively.

(iii) Write down the coordinates of B', C' and D'.

(iv) Join the points A, B, C, D, D', C', B', A in order and give a name to the closed figure ABCDD'C'B'.
Ans.

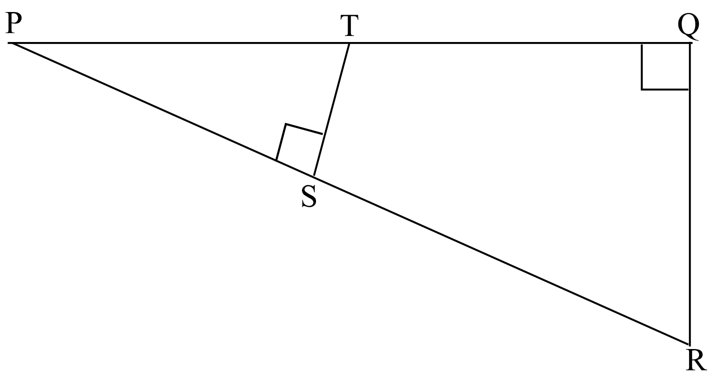


Question 6

(a) In the given figure ∠PQR = ∠PST = 90^0, PQ = 5 cm and PS = 2 cm. **[3]**

(i) Prove that △PQR = △PST

(ii) Find Area of △PQR: Area of quadrilateral SRQT.



Ans.

Given, PQ = 5cm and PS = 2cm

(i). Here, in △PQR and △PST

∠PQR = ∠PST = 90^0

And ∠RPQ = ∠SPT [∵ Being common angle]

$\therefore$ ΔPQR ~ ΔPST [Being AA similarity]

So, $\frac{PQ}{PS} = \frac{QR}{ST} = \frac{PR}{PT} = \frac{5}{2}$

$\therefore \frac{ar(\Delta PQR)}{ar(\Delta PST)} = \frac{5^2}{2^2} = \frac{25}{4}$

$\therefore ar(\Delta PST) = \frac{4}{25} ar(\Delta PQR)$..........................$eq(1)$

(ii)

$$\frac{ar(\Delta PQR)}{ar(\Delta PST)} = \frac{5^2}{2^2} = \frac{25}{4}$$

$$\frac{ar(\Delta PQR) - ar(\Delta PST)}{ar(\Delta PST)} = \frac{25-4}{4} = \frac{21}{4}$$

$$\frac{ar(\square SRQT)}{ar(\Delta PST)} = \frac{21}{4}$$

$$\frac{ar(\square SRQT)}{\frac{4}{25} ar(\Delta PQR)} = \frac{21}{4} \text{.....................} \textit{From equation } 1$$

$$\frac{ar(\square SRQT)}{ar(\Delta PQR)} = \frac{21}{4} \times \frac{4}{25}$$

$$\frac{ar(\square SRQT)}{ar(\Delta PQR)} = \frac{21}{25}$$

(b) The first and last term of a Geometrical Progression (G.P.) are 3 and 96 respectively. If the common ratio is 2, find: [3]

(i) ‘n’ the number of terms of the G.P.

(ii) Sum of the n terms.

Ans. Given first term (a) = 3,last term (l) =96 and common difference (r) = 2.

(i). We know, formula to find nth term of a G.P is $a_n = ar^{n-1}$ or $l = ar^{n-1}$

So, according to question,

$96 = 3 \times 2^{n-1}$

$\Rightarrow 2^{n-1} = 32$

$\Rightarrow 2^{n-1} = (2)^5$

$\Rightarrow n - 1 = 5$

$\therefore n = 6$

(ii). We have, n= 6, r= 2 and a=3

Formula for the sum of n terms of a G.P $(S_n) = \dfrac{a(r^n - 1)}{r - 1}$

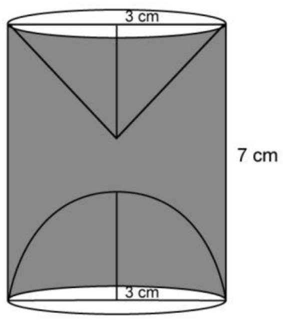
$\therefore$ Sum of 6 terms $(S_6) = \dfrac{3(2^6 - 1)}{2 - 1}$

$$= \frac{3(64-1)}{1}$$

$$= 3 \times 63 = 189$$

(c) A hemispherical and a conical hole is scooped out of a solid wooden cylinder. Find the volume of the remaining solid where the measurements are as follows: The height of the solid cylinder is 7 cm, radius of each of hemisphere, cone and cylinder is 3 cm. Height of cone is 3 cm. Give your answer correct to the nearest whole number Take

$\pi = \dfrac{22}{7}$ **[4]**



Ans.

Given, Height of cylinder (h) = 7cm

Radius (r) = 3cm

$\therefore$ Volume of cylinder $(V_1) = \pi r^2 h$

$= \pi(3)^2 7 = \pi.63cm^3$

Height of cone (H) = 3cm

$\therefore$ Volume of cone (V_2) = $\frac{1}{3}\pi r^2 H$

$= \frac{1}{3}\pi(3)^2 3 = \pi 9cm^3$

Volume of hemisphere (V_3) = $\frac{2}{3}\pi r^3$

$= \frac{2}{3}\pi(3)^3 = \pi 18cm^3$

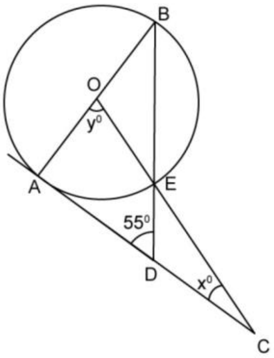
According to question,

Volume of remaining solid = V_1-V_2-V_3

$\therefore$ Remaining volume =
$$\begin{aligned}&\pi.63 - \pi 9 - \pi 18\\ &= \pi(63 - 9 - 18)\\ &= \frac{22}{7} \times 36\\ &= 113.14cm^3\end{aligned}$$

Question 7

(a) In the given figure AC is a tangent to the circle with centre O. If ADB = 55^0 **, find x and y, Give reasons for your answer. [3]**



Ans.

Given that, AC is a tangent to the circle with centre O

And $\angle$ADB= 55^0

Here, ΔABD is a right angled triangle, so $\angle BAD = 90^0$

We know,

Sum of interior angles of a triangle = 180^0

$\angle \text{ADB} + \angle BAD + \angle ABD = 180^0$

$\Rightarrow 55^0 + 90^0 + \angle ABD = 180^0$

$\Rightarrow \angle ABD = 180^0 - 145$

$\therefore \angle ABD = 35^0$

Since, $\angle AOE$ is subtended at the centre and $\angle EBA$ on the circle by the arc AE,

Thus, 2 $\angle ABD = \angle AOE$

$\angle AOE$ = 2×35

$\therefore$ y = 70 ^{0}

In ΔAOC, $\angle OAC = 90^0$

$\because$ Sum of interior angles of triangle = 180 ^{0}

$\therefore \angle OAC + \angle AOC + \angle ACO = 180^0$

$\Rightarrow 90^0 + y + x = 180^0$

$\Rightarrow x + y = 90^0$

$\Rightarrow x - 35^0 = 90$

$\therefore x = 55^0$

(b) The model of a building is constructed with scale factor 1: 30. [3]

(i) If the height of the model is 80 cm, find the actual height of the building in meters.

(ii) If the actual volume of a tank at the top of the building is $27m^3$**, find the volume of the tank on the top of the model.**

Ans. Given scale factor is 1:30

(i). The ratio of height of the model and building is 1:30

So, if height of the model is 80cm

Then, the actual height of the building = 80×30= 2400 cm = 24m

(i)i. Given, actual volume of tank = $27m^3$

Given scale factor =1:30

Here,

1m = $\frac{10}{3}$cm

(1m)^3 = $\left(\frac{10}{3}\right)^3$

1m^3 = $\frac{1000}{27}cm^3$

Then, $27m^3$ = $\frac{1000}{27} \times 27 = 1000cm^3$

Now,

$1cm^3 = 0.001 litres$

$\therefore 1000cm^3 = 0.001 \times 1000 = 1l$itres

Therefore, the required volume of model tank is 1litres.

(c)Given, $\begin{vmatrix} 4 & 2 \\ -1 & 1 \end{vmatrix}$M=6I **, where M is a matrix and 1 is unit matrix of order 2 x 2. [4]**

(i) State the order of matrix M.

(ii) Find the matrix M

Ans.

(i) $\begin{vmatrix} 4 & 2 \\ -1 & 1 \end{vmatrix}$M=6I

For matrix multiplication, the number of columns in first matrix should be equal to the number of rows in the other matrix and the resulting matrix will have order as below:

$$| \ |_{m\times n} \times | \ |_{n\times q} = | \ |_{m\times q}$$

As given:

$$\begin{vmatrix} 4 & 2 \\ -1 & 1 \end{vmatrix} \text{M} = \begin{vmatrix} 6 & 0 \\ 0 & 6 \end{vmatrix}$$

So, the order of M should be 2 x 2

Let M = $\begin{vmatrix} p & q \\ r & s \end{vmatrix}$

$$\begin{vmatrix} 4 & 2 \\ -1 & 1 \end{vmatrix} \text{M} = \begin{vmatrix} 4 & 2 \\ -1 & 1 \end{vmatrix} \times \begin{vmatrix} p & q \\ r & s \end{vmatrix} = \begin{vmatrix} 4p+2r & 4q+2s \\ -p+r & -q+s \end{vmatrix}$$

$$\Rightarrow \begin{vmatrix} 4p+2r & 4q+2s \\ -p+r & -q+s \end{vmatrix} = \begin{vmatrix} 6 & 0 \\ 0 & 6 \end{vmatrix}$$

$4p+2r=6.................eq1$

$-p+r=0.................eq2$

$4q+2s=0.................eq3$

$-q+s=6.................eq4$

From eq2 we get,

$r=p$

Substituting the value of r in eq1

$4r+2r=6$

$6r=6$

$r=1$

$\Rightarrow\ p=1$

From eq4 we get,

$s=6+q$

Substituting the value of r in eq3

$4q+2(6+q)=0$

$4q+12+2q=0$

$6q=-12$

$q=-2$

$\Rightarrow s=6-2=4$

Therefore, M = $\begin{vmatrix} 1 & -2 \\ 1 & 4 \end{vmatrix}$

Question 8

(a) The sum of the first three terms of an Arithmetic Progression (A.P.) is 42 and the product of the first and third term is 52. Find the first term and the common difference. [3]

Ans. Let the first term of A.P be a and the common difference be d.

So, the first three terms could be a-d, a, and a+d.

According to question,

$a-d+a+a+d = 42$

$\Rightarrow 3a = 42$

$\therefore a = \frac{42}{3} = 14$..........(1)

And,

$(a-d)(a+d) = 52$

$\Rightarrow a^2 - d^2 = 52$ $\quad$ $[\because (a+b)(a-b) = a^2 - b^2]$

Putting value of a from (1), we get

$\Rightarrow (14)^2 - d^2 = 52$

$\Rightarrow 196 - 52 = d^2$

$\Rightarrow d^2 = 144$

$\therefore d = 12$

(b) The vertices of a △ABC are A(3, 8), B(-1, 2) and C(6, -6), Find: [3]

(i) Slope of BC.

(ii) Equation of a line perpendicular to BC and passing through A.

Ans. Given, the vertices of triangle ABC are A(3, 8), B(-1, 2) and C(6, -6)

i.e. know, slope of line = $\frac{y_2 - y_1}{x_2 - x_1}$

$\therefore$ Slope of BC = $\frac{-6-2}{6-(-1)} = \frac{-8}{7}$

ii. Let AE be the line which is perpendicular to BC and passes through A

So, Slope of AE = $-\frac{1}{\text{Slope of BC}}$ $[\because$ AE is perpendicular to BC$]$

$\therefore$ Slope of AE = $-\left(\frac{1}{-\frac{8}{7}}\right) = \frac{7}{8}$

Now, we have

Slope of AE (m) = $\frac{7}{8}$ and A (3,8)

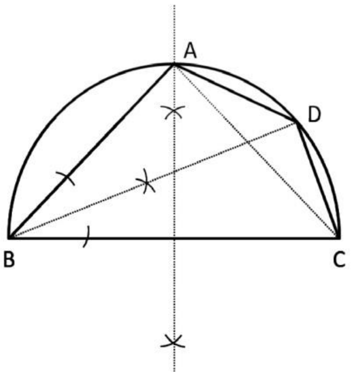
$\therefore$ Equation of line AE

$$= y-8=\frac{7}{8}(x-3)$$
$$\Rightarrow 8y-64=7x-21$$
$$\Rightarrow 7x-8y-21+64=0$$
$$\Rightarrow 7x-8y+43=0$$

(c) Using ruler and a compass only construct a semi-circle with diameter BC = 7cm. Locate a point A on the circumference of the semicircle such that A is equidistant from B and C. Complete the cyclic quadrilateral ABCD, such that D is equidistant from AB and BC. Measure ∠ADC and write it down. [4]

Ans.

1) Construct a semi-circle with diameter BC = 7 cm i.e. radius 3.5 cm
2) Draw the perpendicular bisector of BC and extend it to touch the semi-circle
3) Mark this point as A (Since A should be equidistant from B and C)
4) Join AB and AC
5) Draw the angle bisector of angle ABC and extend it to meet the semi-circle
6) Mark this point as D (D is equidistant from AB and BC)
7) Join AD and CD
8) Measure of $\angle ADC$ is 135^0



Question 9

(a) The data on the number of patients attending a hospital in a month are given below. Find the average (mean) number of patients attending the hospital in a month by using the shortcut method. [3]

Take the assumed mean as 45. Give your answer correct to 2 decimal places.

Number of patients	**10-20**	**20-30**	**30-40**	**40-50**	**50-60**	**60-70**
Number of days	**5**	**2**	**7**	**9**	**2**	**5**

Ans. Given, Assumed mean (A) = 45

The frequency table can be obtained using the data above:

Number of patients	Mid-point(x)	Number of days(f)	d = x-A =x- 45	fd
10-20	15	5	-30	-150
20-30	25	2	-20	-40
30-40	35	7	-10	-70
40-50	45	9	0	0
50-60	55	2	10	20
60-70	65	5	20	100
		$\sum f = 30$		$\sum fd = -140$

We have formula for mean = $A + \dfrac{\sum fd}{\sum f}$

$\therefore$ Mean number of patients attending the hospital in a day

$= 45 + \dfrac{-140}{30} = 45 - 4.667 = 40.33$

Hence, the mean or average number of patients attending the hospital in a month

$= 40.33 \times 30$

$= 1209.9$

(b) Using properties of proportion solve for x, given. [3]

$$\frac{\sqrt{5x} + \sqrt{2x-6}}{\sqrt{5x} - \sqrt{2x-6}} = 4$$

Ans. Given, $\dfrac{\sqrt{5x} + \sqrt{2x-6}}{\sqrt{5x} - \sqrt{2x-6}} = 4$

Using rationalization, we get

$$\frac{\sqrt{5x}+\sqrt{2x-6}}{\sqrt{5x}-\sqrt{2x-6}} \times \frac{\sqrt{5x}+\sqrt{2x-6}}{\sqrt{5x}+\sqrt{2x-6}} = 4$$

$$\Rightarrow \frac{\left(\sqrt{5x}\right)^2+\left(\sqrt{2x-6}\right)^2}{\left(\sqrt{5x}\right)^2-\left(\sqrt{2x-6}\right)^2} = 4$$

$$\Rightarrow \frac{5x+2\sqrt{5x}\sqrt{2x-6}+2x-6}{5x-2x+6} = 4$$

$$\Rightarrow 7x+2\sqrt{5x}\sqrt{2x-6}-6 = 4(3x+6)$$

$$\Rightarrow 7x+2\sqrt{5x}\sqrt{2x-6}-6 = 12x+24$$

$$\Rightarrow 2\sqrt{5x}\sqrt{2x-6} = 5x+30$$

Squaring both the sides,we get

$$\left(2\sqrt{5x}\sqrt{2x-6}\right)^2 = \left(5x+30\right)^2$$

$$\Rightarrow 4\times 5x(2x-6) = 25x^2+300x+900$$

$$\Rightarrow 20x(2x-6) = 25x^2+300x+900$$

$$\Rightarrow 40x^2-120x-25x^2-300x = 900$$

$$\Rightarrow 15x^2-420x-900 = 0$$

$$\Rightarrow x^2-28x-60 = 0$$

$$\Rightarrow x^2-30x+2x-60 = 0$$

$$\Rightarrow x(x-30)+2(x-30) = 0$$

$$\Rightarrow (x-30)(x+2) = 0$$

$$\therefore x = 30 \text{ or } -2$$

(c) Sachin invests Rs. 8500 in 10%, Rs. 100 shares at Rs. 170 He sells the shares when the price of each share rises by Rs. 30. He invests the proceeds in 12% Rs. 100 shares at Rs. 125, Find:[4]

(i) The sale proceeds

(ii) The number of Rs. 125 shares he buys.

(iii) The change in his annual income.

Ans.

Purchase price of each share = Rs. 170

Then, number shares purchased for Rs.8500 = $\frac{8500}{170} = 50$

As the price of share increases by Rs. 30,

So,

Selling price of each share = Rs. (170 +30) = Rs.200

i. Sale proceeds shares = Rs. (200 × 50) = Rs. 10000

ii. The number of shares purchased at Rs. 125 each = $\frac{10000}{125} = 80$

iii. Initially,

Total face value of share = Face value of each share× numbers of share

= 100× 50= Rs.5000

Dividend = 10% of total face value

= $\frac{10}{100} \times 5000$ = Rs.500

After the price of share rises by Rs.30,

Total face value of share = 100×80= Rs. 8000

And, Dividend = 12% of 8000 = $\frac{12}{100} \times 8000 = Rs.960$

Hence, the change in annual income = Rs.(960-500) =Rs. 460

Question 10

(a) Use graph paper for this question. The marks obtained by 120 students in an English test are given below. [6]

Marks	**0-10**	**10-20**	**20-30**	**30-40**	**40-50**	**50-60**	**60-70**	**70-80**	**80-90**	**90-100**
No. of students	**5**	**9**	**16**	**22**	**26**	**18**	**11**	**6**	**4**	**3**

Draw the ogive and hence, estimate:

(i) The median marks.

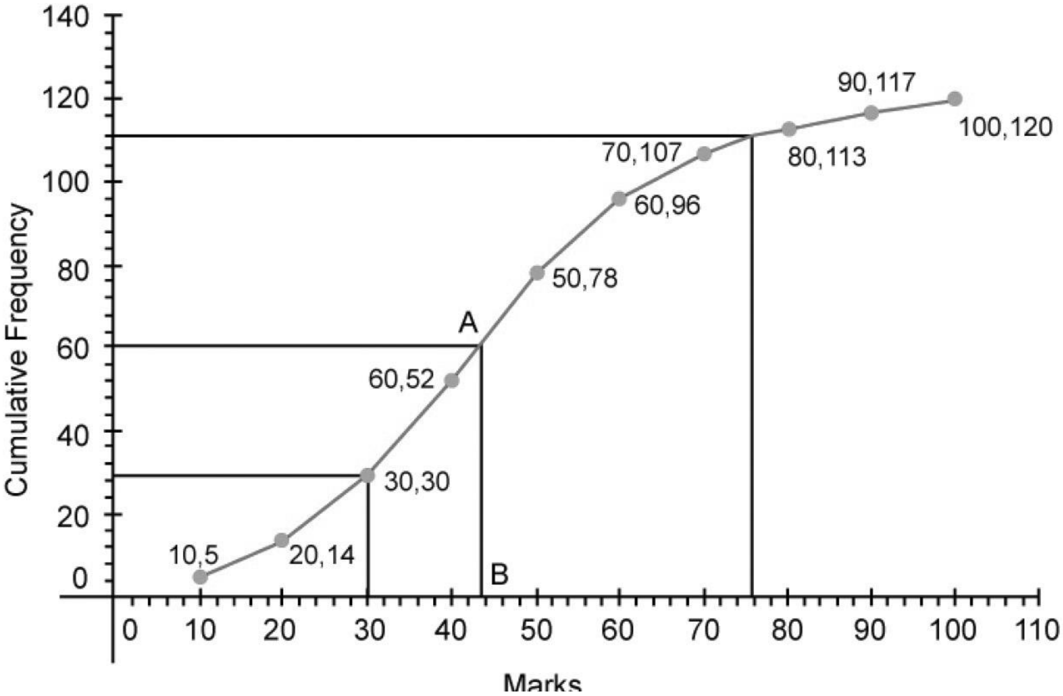
(ii) The number of students who did not pass the test if the pass percentage was 50.

(iii) The upper quartile marks.

Ans. We have to prepare a frequency table first,

Class Interval	Frequency	Cumulative frequency
0-10	5	5
10-20	9	14
20-30	16	30
30-40	22	52
40-50	26	78
50-60	18	96
60-70	11	107
70-80	6	113
80-90	4	117
90-100	3	120

Here, n =120 and $\frac{n}{2} = \frac{120}{2} = 60$



i. Through mark 60, draw a line segment parallel to x-axis which meets the curve at A. From A, draw a line parallel to y- axis which meets the x-axis at point B. Therefore,

Median = 43

ii. Number of students who did not pass the test = the students who obtained less than 50 % marks =5+9+16+22+26=78

iii. The formula for upper quartile = $\left(\frac{3n}{4}\right)^{th}$ term.

Therefore, the upper quartile marks = $\left(\frac{3\times 120}{4}\right)^{th}$ term = 90^{th} term= 117

(b) A man observes the angle of elevation of the top of the lower to be 45^0. He walks towards it in a horizontal line through its base. On covering 20 m the angle of elevation changes to 60^0. Find the height of the tower correct to 2 significant figures. [4]

Ans.

Let PQ be the height of the tower and R and S be two positions of man from where the angle of elevation was formed.

According to question, RS = 20m, $\angle QSP = 45^0$ and $\angle QRP = 60^0$

Let PQ = h and RP = x

Then, in ΔQSP,

$$\tan 45^0 = \frac{PQ}{SP}$$

$$\Rightarrow \tan 45^0 = \frac{h}{20+x} \qquad [\because \text{Being right angled triangle \& } \tan 45^0 = 1]$$

$$\Rightarrow 20 + x = h$$

$$\Rightarrow x = h - 20$$

Similarly, in ΔPQR

$$\tan 60^0 = \frac{PQ}{RP}$$

$$\Rightarrow \sqrt{3} = \frac{h}{x}$$

$$\Rightarrow x = \frac{h}{\sqrt{3}}$$

Now, we can write, $h - 20 = \frac{h}{\sqrt{3}}$ [Combining the above equations]

$\Rightarrow \sqrt{3}(h-20)=h$
$\Rightarrow \sqrt{3}h-20\sqrt{3}=h$
$\Rightarrow \sqrt{3}h-h=20\sqrt{3}$
$\Rightarrow h(\sqrt{3}-1)=20\sqrt{3}$
$\Rightarrow h=\dfrac{20\sqrt{3}}{(\sqrt{3}-1)}$
$\therefore h=47.32$

Hence, the height of the tower is 47.32m.

Question 11

(a) Using the Remainder Theorem find the remainders obtained when
$x^3 + (kx + 8)x + k$ **is divided by x + 1 and x – 2.Hence find k if the sum of the two remainders is 1.** **[3]**

Ans.

Let P(x) be a polynomial

So, we have P(x) = $x^3 + (kx + 8)x + k$

When P(x) is divided by (x+1) and (x-2), the remainder is P(-1) and P(2) respectively.

Now, P(-1) $\begin{aligned}&=(-1)^3+[k(-1)+8]-1+k\\&=-1+(k-8)+k\\&=2k-9\end{aligned}$

And,

P(2) $\begin{aligned}&=(2)^3+[k(2)+8]2+k\\&=8+(2k+8)2+k\\&=8+4k+16+k\\&=5k+24\end{aligned}$

According to question, the sum of the two remainders is 1

i.e.

$P(-1)+P(2)=1$

$\Rightarrow$(2k-9) + (5k+24) =1

$\Rightarrow$2k-9+5k+24=1

$\Rightarrow$7k+15=1

$\Rightarrow$7k = -14

$\therefore k=-2$

(b) The product of two consecutive natural numbers which are multiples of 3 is equal to 810. Find the two numbers. [3]

Ans. Let the two consecutive natural numbers which are multiple of 3 be 3x and (3x+3)

Then, according to question,

Product of the numbers = 810

$$\Rightarrow 3x\times(3x+3)=810$$
$$\Rightarrow 9x^2+9x=810$$
$$\Rightarrow x^2+x=90$$
$$\Rightarrow x^2+x-90=0$$
$$\Rightarrow x^2+10x-9x+90=0$$
$$\Rightarrow x(x+10)-9(x+10)=0$$
$$\Rightarrow (x+10)(x-9)=0$$
$$\therefore x=-10 \text{or } 9$$

Taking x=9, we get 3x = 3×9 = 27

And (3x+3) = (3×9+3) = 30

Taking x= -10, we get 3x= 3×-10 = −30

And, (3x+3) = [3×(-10)+3]=−27

Hence, the required numbers are 27 and 30 or −27 and −30

(c) In the given figure, ABCDE is a pentagon inscribed in a circle such that AC is a diameter and side BC||AE. If ∠BAC = 50°, find giving reasons: [4]

(i) ∠ACB

(ii) ∠EDC

(iii) ∠BEC

Hence prove that BE is also a diameter.

Ans. Given that, ABCDE is a pentagon inscribed in a circle such that AC is the diameter of circle and BC||AE.

And , $\angle BAC = 50^0$

Here, $\triangle ABC$ is right angled triangle , so $\angle ABC = 90^0$

Now,

$\angle ABC + \angle BAC + \angle ACB = 180^0$

$\Rightarrow 50^0 + 90^0 + \angle ACB = 180$ [Sum of interior angles of triangle $=180^0$]

$\Rightarrow \angle ACB = 180^0 - 140^0$

$\therefore \angle ACB = 40^0$

Since, BC||AE

$\therefore \angle ACB = \angle EAC = 40^0$ [Being alternate angle]

Again, in $\Delta AEC, \angle AEC = 90^0$

[Sum of interior angles of triangle $=180^0$]

$\angle AEC + \angle EAC + \angle ACE = 180^0$

$\Rightarrow 90^0 + 40^0 + \angle ACE = 180$

$\Rightarrow \angle ACE = 180^0 - 130^0$

$\therefore \angle ACE = 50^0$

And, $\angle BAC = 50^0$

Therefore, AB||CE

[$\because$ Being alternate angles made by transversal AC with lines CE&AB,equal]

Now, $\angle BAE = 50^0 + 40^0 = 90^0$

Also, since $\angle$EAB is subtended by EB on the circle
So,BE is a diameter of the given circle.

Here, AEDC is cyclic quadrilateral
So,
$\angle$EAC + $\angle$ EDC = 180^0
$\Rightarrow 40^0 + \angle$ EDC = 180^0
$\therefore \angle$ EDC = 140^0

Now,
We see that Arc BC subtends angles $\angle$BAC and $\angle$BEC on the same side of the circle
Therefore,
$\angle$BAC = $\angle$BEC = 50^0

ICSE 2020

Grade 10

Mathematics

Time: 2 ½ Hours **Max. Marks: 80**

Answers to this Paper must be written on the paper provided separately. You will not be allowed to write during the first 15 minutes. This time is to be spent in reading the question paper. The time given at the head of this Paper is the time allowed for writing the answers.

Attempt all questions from Section A and any four questions from Section B.

All working, including rough work, must be clearly shown and must be done on the same sheet as the rest of the answer. Omission of essential working will result in loss of marks. The intended marks for questions or parts of questions are given in brackets [J. Mathematical tables are provided.

SECTION A

Attempt all questions from this Section.

Question 1

(a) Solve the following Quadratic Equation: **3**

$x^2 - 7x + 3 = 0$

Give your answer correct to two decimal places.

(b) Given A $A = \begin{bmatrix} x & 3 \\ y & 3 \end{bmatrix}$ **3**

If 42 = 31, where I is the identity matrix of order 2, find x and y.

(c) Using ruler and compass construct a triangle ABC where AB = 3 cm, BC = 4 cm and $\angle ABC = 90^0$. Hence construct a circle circumscribing the triangle ABC. Measure and write down the radius of the circle. **4**

Question 2

(a) Use factor theorem to factories $6x^3+17x^2+4x-12$ completely. **3**

(b) Solve the following inequation and represent the solution set the number line

$\frac{3x}{5}+2<x+4\leq\frac{x}{2}+5x\in R$ **3**

(c) Draw a Histogram for the give data, using a graph paper:

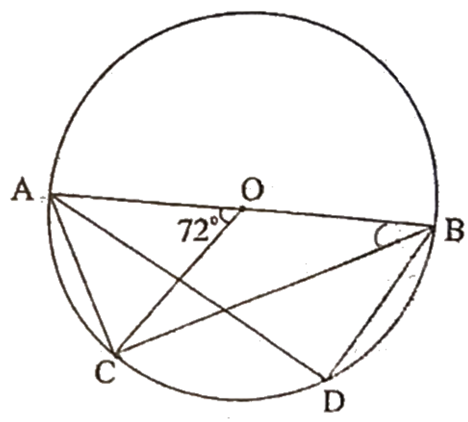
Weekly Wages (in₹)	No. of People
3000-4000	4
4000-5000	9
5000-6000	18
6000-7000	6
7000-8000	7
8000-9000	2
9000-10000	4

Estimate the mode from the graph **4**

Question 3

(a) In the figure given bellow, O is the center of the circle and AB is a diameter. **3**

If AC = BD and $\angle$AOC = 72° Find:



(i) $\angle$ABC

(ii) $\angle$BAD

(iii) $\angle$ABD

(b) Prove that: **3**

$$\frac{\sin A}{1+\cot A}-\frac{\cos A}{1+\tan A}=\sin A-\cos A$$

(c) In what ratio is the line joining P(5, 3) and Q(-5, 3) divided by the y-axis? Also find coordinates of the point of intersection. **4**

Question 4

(a) A solid spherical ball of radius 6 cm is melted and recast into 64 identical spherical marbles. Find the radius of each marble, **3**

(b) Each of the letters of the word `AUTHORIZES' is written on identical circular discs and put in a bag. They are well shuffled. If a disc is drawn at random from the bag, what is the probability that the letter is: **3**

(i) a vowel

(ii) one of the first 9 letters of the English alphabet which appears in the it given word

(iii) one of the last 9 letters of the English alphabet which appears in the in word?

(c) Mr. Bedi visits the market and buys the following articles: **4**

Medicines costing ₹ 950, GST @ 5%

A pair of shoes costing ₹ 3000, GST @ 18%

A Laptop bag costing ₹ 1000 with a discount of 30%, GST @, 18%,

(i) Calculate the total amount of GST paid.

(ii) The total bill amount including GST paid by Mr. Bedi,

SECTION B

Attempt any four question from this section

Question 5

(a) A company with 500 shares of nominal value 120 declares an annual dividend ut 131 15%. Calculate:
(i) the total amount of dividend paid by the company. **3**

(ii) annual income of Mr. Sharma who holds 80 shares of the company

If the return percent of Mr. Sharma from his shares is 10%, find the Markel value of each share.

(b) The mean of the following data is 16 Calculate the value of f **3**

Marks	5	10	15	20	25
No. of Students	3	7	f	9	6

(c) The 4th, 6th and the last term of a geometric progression are 10, 40 and 640 respectively. If the common ratio is positive, find the first term, common ratio number of terms of the series. **4**

Question 6

(a) If $A = \begin{bmatrix} 3 & 0 \\ 5 & 1 \end{bmatrix} and B = \begin{bmatrix} -4 & 2 \\ 1 & 0 \end{bmatrix}$ **3**

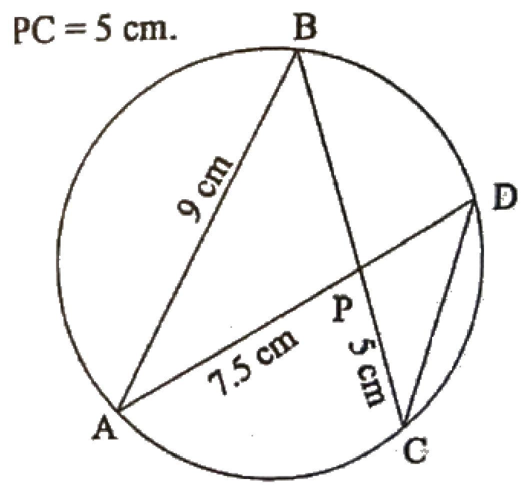
Find $A^2 - 2AB + b^2$

(b) In the given figure AB = 9cm, PA = 7.5 and PC = 5Cm. Chords AD and BC intersect at P.

(i) Prove that Δ PAB ~ ΔPCD

(ii) Find the length of CD

(iii) Find area of ΔPAB: area of ΔPCD **3**



(c) From the top of a cliff, the angle of depression of the top and bottom of a tower are observed to be 45° and 60° respectively. Uf the height of the tower is 20m. **4**

Find:

(i) the height of the cliff

(ii) the distance between the cliff and the tower.

Question 7

(a) Find the value of 'p' if the lines, 5x — 3y + 2 0 and 6x — py + 7 = 0 are perpendicular to each other. Hence find the equation of a line passing through (-2, —1) and parallel to 6x — py + 7 = 0. **3**

(b) Using properties of find x : y, given:

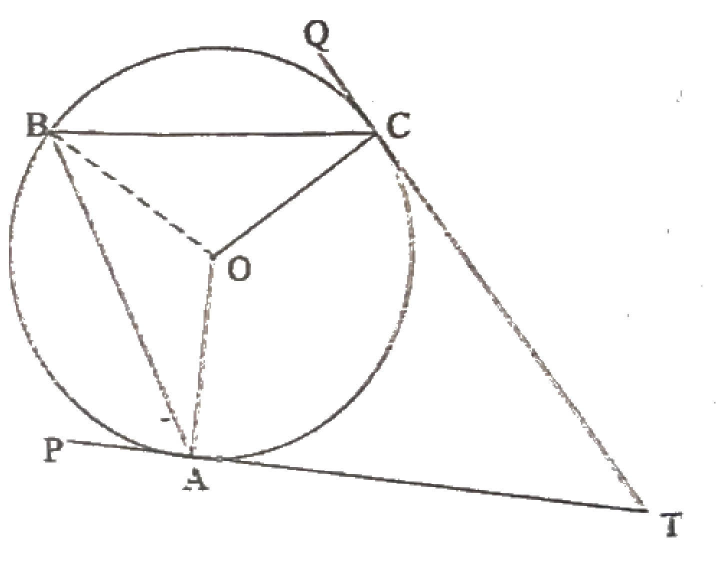
$$\frac{x^2+2x}{2x+4}=\frac{y^2+3y}{3y+9}$$ **3**

(c) In the given figure TP and TQ are two tangents to the circle with center O, touching at A and C respectively, If $\angle$BCQ = 55° and $\angle$BAP = 60°, find:

(i) $\angle$OBA and $\angle$OBC **4**

(ii) $\angle$AOC

(iii) $\angle$ATC



Question 8

(a) What must e added to the polynomial 2x3-3x2-8x, so that it leaves a remainder 10 when divided by 2x + 1? **3**

(b) Mr. Sonu has a recurring deposit account and deposits ₹ 750 per month for 2 years If he gets ₹ 19125 at the time of maturity, find the rate of interest. **3**

(c) Use graph paper for this question.
Take 1 cm = 1 unit on both x and y axes **4**

(i) Plot the following points on your graph sheets:
A(-4, 0), B(-3, 2), C(0, 4) D(4, 1) and E(7, 3)

(ii) Reflected the point B,C,D and E on the x-axis and name then as B,C,D and E respectively.

(iii) Join the points A,B,C,D,E,E,D,C,B and A in order.

(iv) Name the close figure farmed

Question 9

(a) 40 student enter for a game of shot-put competition.
The distance thrown (in meters) is recorded below: **6**

Distance In m	12-13	13-14	14-15	15-16	16-17	17-18	18-19
Number Of Students	3	9	12	9	4	2	1

Use a graph paper to draw an ogive for the above distribution. Use a scale of 2 cm = 1 m on one axis and 2 cm = 5 students on the other axis. Hence using your graph find:

(i) the median

(ii) Upper Quartile

(iii) number of student who cover a distance which is above $16\frac{1}{2}$ m.

(a) If $x = \frac{\sqrt{2a+1}+\sqrt{2a-1}}{\sqrt{2a+1}-\sqrt{2a-1}}$, prove that $x^2 - 4ax + 1 = 0$ **4**

Question 10

(a) If the 6th of an A.P. is equal to four times its first term and the sum of first six terms is 75, find the first term and the common difference. **3**

(b) The difference of two natural numbers is 7 and their product is 450. Find the numbers. **3**

(c) Use ruler and compass for this question. Construct a circle of radius 4.5 cm. **3**

Draw a chord. AB = 6cm.

(i) Find the locus of points equidistant from A and B. Mark the point where it meets the circle as D.

(ii) Join AD and find the locus of points which are equidistant from AD and AB. Murk the point where it meets the circle as C.

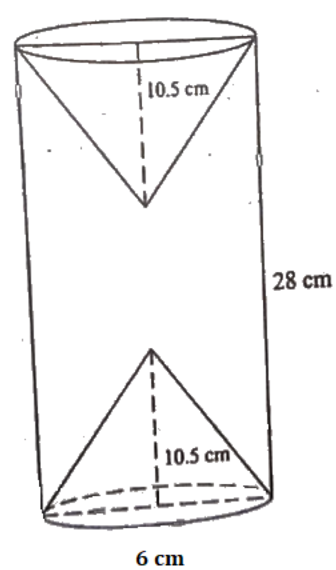
(iii) Join and CD. Measure and write down the length of side CD of the quadrilateral ABCD.

Question 11

(a) A model of a high rise building is made to a scale of 1: 50.

(i) If the height of the model is 0.8 m, find the height of the actual building.
(ii) If the floor area of a flat in the building is 20 m^2, find the floor area of that in the model. **3**

(b) From a solid wooden cylinder of height 28 cm and diameter 6 cm, two conical cavities are hollowed out. The diameters of the cones are also of 6 cm and height 10.5 cm. Taking $\pi = \frac{22}{7}$ find the volume of the remaining solid. **3**



(c) Prove the identity **4**

$$\left(\frac{1-\tan\theta}{1-\cot\theta}\right)^2 = \tan^2\theta$$

ICSE Board
Class X Mathematics
Board Paper
Semester 1 – 2021

Time: 90 minutes **Marks: 40**

Maximum Marks: 40
Time allowed One and a half hours
You will not be allowed to write during the first 10 minutes
This time is to be spent in reading the question paper.
ALL QUESTIONS ARE COMPULSORY.
The marks intended for questions are given in brackets [ ]

Select the correct option for each of the following questions.

Question 1

If (x + 2) is a factor of the polynomial $x^3 - kx^2 - 5x + 6$ then the value of k is: [1]

(a) 1
(b) 2
(c) 3
(d) -2

Question 2

The solution set of the inequation $x - 3 \geq -5, x \in R$ is: [1]

(a) $\{x: x > -2, x \in R\}$
(b) $\{x: x \leq -2, x \in R\}$
(c) $\{x: x \geq -2, x \in R\}$
(d) $\{-2, -1, 0, 1, 2\}$

Question 3

The product AB of two matrices A and B is possible if: [1]

(a) A and B have the same number of rows.
(b) The number of columns of A is equal to the number of rows of B.
(c) The number of rows of A is equal to the number of columns of B
(d) A and B have the same number of columns

Question 4

If 70, 75, 80, 85 are the first four terms of an Arithmetic Progression. Then the 10^{th} term is:[1]

(a) 35
(b) 25
(c) 115
(d) 105

Question 5

The selling price of a shirt excluding GST is Rs. 800. If the rate of GST is 12% then the total price of the shirt is: [1]

(a) Rs. 704

(b) Rs. 96

(c) Rs. 896

(d) Rs. 848

Question 6

Which of the following quadratic equations has 2 end 3 as its roots? [1]

(a) $x^2 - 5x + 6 = 0$

(b) $x^2 + 5x + 6 = 0$

(c) $x^2 - 5x - 6 = 0$

(d) $x^2 + 5x - 6 = 0$

Question 7

If x, 5.4, 5, 9 are in proportion then x is: [1]

(a) 3

(b) 9.72

(c) 25

(d) 25/3

Question 8

Mohit opened a Recurring deposit account in a bank for 2 years. He deposits Rs. 1000 every month and receives Rs. 25500 on maturity. The interest he earned in 2 years is: [1]

(a) Rs. 13500

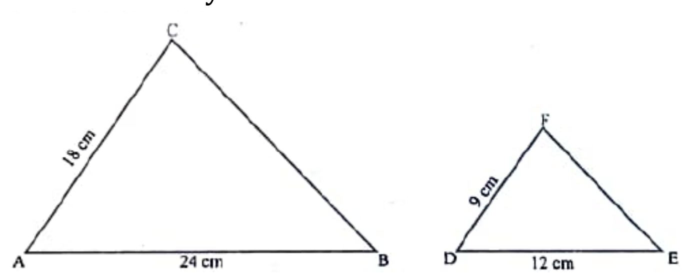
(b) Rs. 3000

(c) Rs. 24000

(d) Rs. 1500

Question 9

In the given figure AB = 24 cm, AC = 18 cm, DE = 12cm, DF = 9 cm and $\angle BAC = \angle EDF$. Then $\Delta ABC \sim \Delta DEF$ by the condition: [1]



(a) AAA

(b) SAS

(c) SSS

(d) AAS

Question 10

If $A = \begin{bmatrix} 5 & 10 \\ 3 & -4 \end{bmatrix}$ and $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ then AI is equal to [1]

(a) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

(b) $\begin{bmatrix} 5 & 10 \\ -3 & 4 \end{bmatrix}$

(c) $\begin{bmatrix} 5 & 10 \\ 3 & -4 \end{bmatrix}$

(d) $\begin{bmatrix} 15 & 15 \\ -1 & -1 \end{bmatrix}$

Question 11

The polynomial $x^3 - 2x^2 + ax + 12$ when divided by $(x + 1)$ leaves a remainder 20, then 'a' is equal to: [1]

(a) - 31
(b) 9
(c) 11
(d) - 11

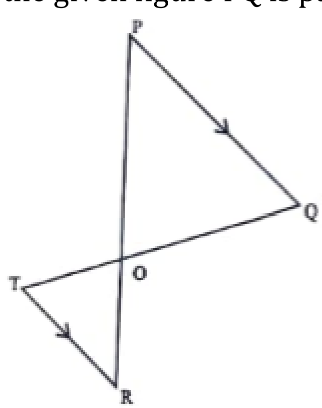
Question 12

In an Arithmetic Progression (A.P.) if, first term is 5, common difference is - 3 and then n^{th} term is - 7, then n is equal to: [1]

(a) 5
(b) 17
(c) - 13
(d) 7

Question 13

In the given figure PQ is parallel to TR, then by using condition of similarity: [1]



(a) $\frac{PQ}{RT} = \frac{OP}{OT} = \frac{OQ}{OR}$

(b) $\frac{PQ}{RT} = \frac{OP}{OR} = \frac{OQ}{OT}$

(c) $\frac{PQ}{RT} = \frac{OR}{OP} = \frac{OQ}{OT}$

(d) $\frac{PQ}{RT} = \frac{OP}{OR} = \frac{OT}{OQ}$

Question 14

If a, b, c, and d are proportional then $\frac{a + b}{a - b}$ is equal to: [1]

(a) $\frac{c}{d}$

(b) $\frac{c - d}{c + d}$

(c) $\frac{d}{c}$

(d) $\frac{c + d}{c - d}$

Question 15

The first four terms of an Arithmetic Progression (A. P.), whose first term is 4 and common difference is -6, are: [1]

(a) 4, -10, -16, -22
(b) 4, 10, 16, 22
(c) 4, -2, -8, -14
(d) 4, 2, 8, 14

Question 16

One of the roots of the quadratic equation $x^2 - 8x + 5 = 0$ is 7.3166. The root of the equation correct to 4 significant figures is: [1]

(a) 7.3166
(b) 7.317
(c) 7.316
(d) 7.32

Question 17

(x + 2) and (x + 3) are two factors of the polynomial $x^3 + 6x^2 + 11x + 6$. If this polynomial is completely factorised the result is: [2]

(a) $(x - 2)(x + 3)(x + 1)$
(b) $(x + 2)(x - 3)(x - 1)$
(c) $(x + 2)(x + 3)(x - 1)$
(d) $(x + 2)(x + 3)(x + 1)$

Question 18

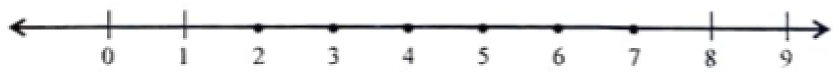
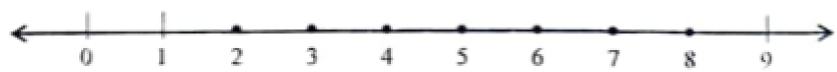
The sum of the first 20 terms of the Arithmetic Progression 2, 4, 6, 8,....is : [2]

(a) 400
(b) 840
(c) 420
(d) 800

Question 19

The solution set on the number line of the linear inequation: [2]

$2y - 6 < y + 2 \leq 2y, y \in N$ is

(a)	
(b)	
(c)	
(d)	

Question 20

If *x, y, z* are in continued proportion then $(y^2 + z^2) : (x^2 + y^2)$ is equal to: [2]

(a) $z : x$
(b) $x : z$
(c) zx
(d) $(y + z):(x + y)$

Question 21

The marked price of an article is Rs. 5,000. The shopkeeper gives a discount of 10%. If the rate of GST is 12%, then the amount paid by the customer including GST is: [2]

(a) Rs. 5040
(b) Rs. 6100
(c) Rs. 6272
(d) Rs. 6160

Question 22

If A = $\begin{bmatrix} 3 & 5 \\ 1 & 4 \end{bmatrix}$, B = $\begin{bmatrix} 2 & 4 \\ 0 & 3 \end{bmatrix}$ and C = $\begin{bmatrix} 1 & -1 \\ 2 & 1 \end{bmatrix}$, then 5A – BC is equal to: [2]

(a) $\begin{bmatrix} -5 & -23 \\ 1 & 17 \end{bmatrix}$

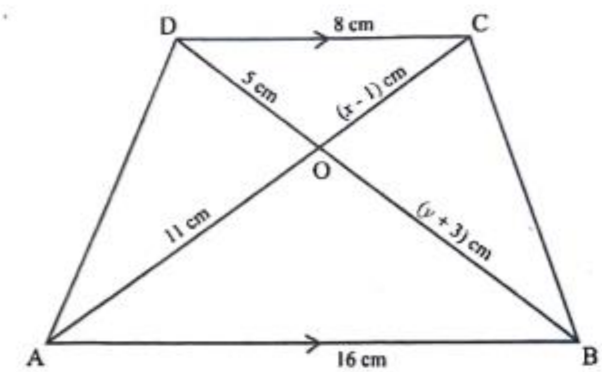
(b) $\begin{bmatrix} 5 & 23 \\ 1 & 17 \end{bmatrix}$

(c) $\begin{bmatrix} -2 & 8 \\ -3 & 3 \end{bmatrix}$

(d) $\begin{bmatrix} 5 & 23 \\ -1 & 17 \end{bmatrix}$

Question 23

In the given figure ABCD is a trapezium in which DC is parallel to AB.
AB = 16 cm and DC = 8 cm. OD = 5 cm, OB = (y + 3) cm, OA = 11 cm and OC = (x – 1) cm.
Using the given information answer the following questions.



i. From the given figure name the pair of similar triangles: [1]

(a) ΔOAB, ΔOBC
(b) ΔCOD, ΔAOB
(c) ΔADB, ΔACB
(d) ΔCOD, ΔCOB

ii. The corresponding proportional sides with respect to the pair of similar triangles obtained in (i): [1]

(a) $\frac{CD}{AB} = \frac{OC}{OA} = \frac{OD}{OB}$

(b) $\frac{AD}{BC} = \frac{OC}{OA} = \frac{OD}{OB}$

(c) $\frac{AD}{BC} = \frac{BD}{AC} = \frac{AB}{DC}$

(d) $\frac{OD}{OB} = \frac{CD}{CB} = \frac{OC}{OA}$

iii. The ratio of the sides of the pair of similar triangles is: [1]

(a) 1 : 3
(b) 1 : 2
(c) 2 : 3
(d) 3 : 1

iv. Using the ratio of sides of the pair of similar triangles the values of x and y are respectively: [1]

(a) x = 4.6, y = 7
(b) x = 7, y = 7
(c) x = 6.5, y = 7
(d) x = 6.5, y = 2

Question 24

Two cars X and Y use 1 litre of diesel to travel x km and (x + 3) km respectively. If both the cars covered a distance of 72 km, then:

i. The number of litres of diesel used by car X is: [1]

(a) $\frac{72}{x-3}$ litres

(b) $\frac{72}{x+3}$ litres

(c) $\frac{72}{x}$ litres

(d) $\frac{12}{x}$ litres

ii. The number of litres of diesel used by car Y is: [1]

(a) $\frac{72}{x-3}$ litres

(b) $\frac{72}{x+3}$ litres

(c) $\frac{72}{x}$ litres

(d) $\frac{12}{x+3}$ litres

iii. If car X used 4 litres of diesel more than car Y in the journey, then: [1]

(a) $\frac{72}{x-3} - \frac{12}{x} = 4$

(b) $\frac{72}{x+3} - \frac{72}{x} = 4$

(c) $\frac{72}{x} - \frac{72}{x+3} = 4$

(d) $\frac{72}{x-3} - \frac{72}{x+3} = 4$

iv. The amount of diesel used by the car X is: [1]

(a) 6 litres
(b) 12 litres
(c) 18 litres
(d) 24 litres

Question 25

Joseph has a recurring deposit account in a bank for two years at the rate of 8% per annum simple interest

i. If at the time of maturity Joseph receives Rs. 2000 as interest then the mostly instalment is: [1]

(a) Rs. 1200
(b) Rs. 600
(c) Rs. 1000
(d) Rs. 1600

ii. The total amount deposited in the bank: [1]

(a) Rs. 25000
(b) Rs. 24000
(c) Rs. 26000
(d) Rs. 23000

iii. The amount Joseph receives on maturity is: [1]

(a) Rs. 27000
(b) Rs. 25000
(c) Rs. 26000
(d) Rs. 28000

iv. If the monthly instalment is Rs. 100 and the rate of interest is 8%, in how many months Joseph will receive Rs. 52 as interest? [1]

(a) 18
(b) 30
(c) 12
(d) 6

Solution

Solution 1

Correct option: (b) 2

If (x + 2) is a factor of the polynomial $x^3 - kx^2 - 5x + 6$, then

$(-2)^3 - k(-2)^2 - 5(-2) + 6 = 0$

$\therefore k = 2$

Solution 2

Correct option: (c) $\{x: x \geq -2, x \in R\}$

$x - 3 \geq -5$

$\therefore x \geq -5 + 3$

$\therefore x \geq -2$

Solution 3

Correct option: (b) The number of columns of A is equal to the number of rows of B.

Solution 4

Correct option: (c) 115

a = 70, d = 5

$\therefore a_{10} = 70 + (9)5 = 115$

Solution 5

Correct option: (c) ₹ 896

SP = ₹800

GST = 12%

Total = 800 + 0.12×800 = ₹896

Solution 6

Correct option: (a) $x^2 - 5x + 6 = 0$

Roots are 2 and 3

Then the quadratic equation will be given by

x^2 – (sum of roots)x – (product of roots)=0

$\therefore x^2 - 5x + 6 = 0$

Solution 7

Correct option: (a) 3

$9 \times x = 5.4 \times 5$

$\therefore x = 3$

Solution 8
Correct option: (d) ₹1500
Total deposit = 1000×24 = ₹24,000
Maturity amount = ₹25,500
$\therefore$Interest = 25,500 – 24,000 = ₹1,500

Solution 9
Correct option: (b) SAS
In ΔABC and ΔDEF
$\frac{AB}{DE} = \frac{AC}{DF} = \frac{2}{1}$
$\angle BAC = \angle EDF$
Hence, $\Delta ABC \sim \Delta DEF$...(SAS test)

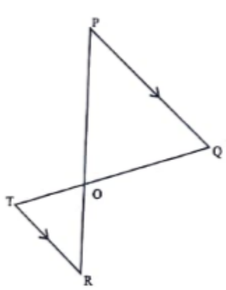
Solution 10
Correct option: (c) $\begin{bmatrix} 5 & 10 \\ 3 & -4 \end{bmatrix}$

AI = A …any matrix multiplied to identity matrix gives the same matrix.

Solution 11
Correct option: (d) -11
By remainder theorem,
$(-1)^3 - 2(-1)^2 + a(-1) + 12 = 20$
$\therefore$ a = -11

Solution 12
Correct option: (a) 5
a = 5, d = -3, a_n = -7
$\therefore$ 5 +(n-1)(-3) = -7
$\therefore$ n = 5

Solution 13
Correct option: (b) $\frac{PQ}{RT} = \frac{OP}{OR} = \frac{OQ}{OT}$



PQ || TR

PR transversal, hence

$\angle P = \angle R \ldots$(alternate angles)

QT transversal, hence

$\angle Q = \angle T \ldots$(alternate angles)

$\therefore \Delta PQO \sim \Delta RTO \ldots$(AA test)

$\therefore \frac{PQ}{RT} = \frac{OP}{OR} = \frac{OQ}{OT} \ldots$(c.p.c.t.)

Solution 14

Correct option: (d) $\frac{c+d}{c-d}$

If a, b, c, and d are proportional then,

$\therefore \frac{a}{b} = \frac{c}{d}$

by componendo-dividendo

$\frac{a+b}{a-b} = \frac{c+d}{c-d}$

Solution 15

Correct option: (c) 4, -2, -8, -14

a = 4 and d = -6, hence the AP is 4, (4-6), (4-6-6), (4-6-6-6), which is 4, -2, -8, -14.

Solution 16

Correct option: (b) 7.317

One of the roots of the quadratic equation $x^2 - 8x + 5 = 0$ is 7.3166.

Correct to 4 significant figures is 7.317.

Solution 17

Correct option: (d) (x+2)(x+3)(x+1)

Checking for (x+1), i.e. x = -1,

$P(x) = x^3 + 6x^2 + 11x + 6$

$P(-1) = (-1)^3 + 6(-1)^2 + 11(-1) + 6 = 0$

Hence (x+1) is the third factor.

Solution 18

Correct option: (c) 420

a = 2, d = 2

$S_{20} = \frac{20}{2}\left(2 \times 2 + (20-1)2\right) = 420$

Solution 19

Correct option: (b)

$2y - 6 < y + 2 \le 2y$

Hence,

$2y - 6 < y + 2$

$\therefore y < 8$

Also,

$y + 2 \le 2y$

$\therefore 2 \le y$

So,

$\therefore 2 \le y < 8$

Solution 20

Correct option: (a) z:x

If x, y, z are in continued proportion then,

$y^2 = xz$

$$\frac{(y^2 + z^2)}{(x^2 + y^2)} = \frac{(xz + z^2)}{(x^2 + xz)} = \frac{z(x + z)}{x(x + z)} = z : x$$

Solution 21

Correct option: (a) ₹5040

Marked price = ₹5,000

Discount = 10%

$\therefore$SP = 5,000×0.90 = ₹4,500

GST = 12%

Final amount paid = 4,500×1.12 = ₹5,040

Solution 22

Correct option: (d) $\begin{bmatrix} 5 & 23 \\ -1 & 17 \end{bmatrix}$

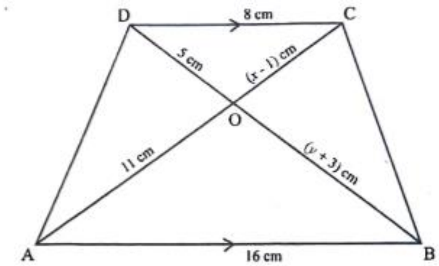
5A – BC

$$= 5\begin{bmatrix} 3 & 5 \\ 1 & 4 \end{bmatrix} - \begin{bmatrix} 2 & 4 \\ 0 & 3 \end{bmatrix}\begin{bmatrix} 1 & -1 \\ 2 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 15 & 25 \\ 5 & 20 \end{bmatrix} - \begin{bmatrix} 10 & 2 \\ 6 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 5 & 23 \\ -1 & 17 \end{bmatrix}$$

Solution 23 (i)
Correct option: (b) ΔCOD, ΔAOB



DC|| AB
Hence we get,
$\angle CDO = \angle ABO$ …(alternate angles)
$\angle DCO = \angle BAO$ …(alternate angles)
$\therefore \Delta COD \sim \Delta AOB$ …(AA test)

Solution 23 (ii)
Correct option: (a) $\frac{CD}{AB} = \frac{OC}{OA} = \frac{OD}{OB}$

$\Delta COD \sim \Delta AOB$

$\therefore \frac{CD}{AB} = \frac{OC}{OA} = \frac{OD}{OB}$ …(c.p.c.t.)

Solution 23 (iii)
Correct option: (b) 1:2

$\frac{CD}{AB} = \frac{OC}{OA} = \frac{OD}{OB} = \frac{8}{16} = \frac{1}{2}$

Solution 23 (iv)
Correct option: (c) x = 6.5, y = 7

$\frac{CD}{AB} = \frac{OC}{OA} = \frac{OD}{OB} = \frac{1}{2}$

$\therefore \frac{x-1}{11} = \frac{5}{y+3} = \frac{1}{2}$

$\therefore \frac{x-1}{11} = \frac{1}{2}$

$\therefore x = 6.5$

Also,

$\frac{5}{y+3} = \frac{1}{2}$

$\therefore y = 7$

Solution 24 (i)

Correct option: (c) $\frac{72}{x}$liters

Car X

x kms = 1 litre

72 kms = $\frac{72}{x}$liters

Solution 24 (ii)

Correct option: (b) $\frac{72}{x+3}$liters

Car y

x+3 kms = 1 litre

72 kms = $\frac{72}{x+3}$liters

Solution 24 (iii)

Correct option: (c) $\frac{72}{x}-\frac{72}{x+3}=4$

Car X used 4 litres of diesel more than car Y, hence

$$\frac{72}{x}-\frac{72}{x+3}=4$$

Solution 24 (iv)

Correct option: (b) 12 litres

$$\frac{72}{x}-\frac{72}{x+3}=4$$

$$\therefore 72x+216-72x=4x^2+12x$$

$$\therefore 4x^2+12x-216=0$$

$$\therefore x^2+3x-54=0$$

$$\therefore (x-6)(x+9)=0$$

$$\therefore x=6...(\text{km can't be negative})$$

$$\therefore \frac{72}{x}=12 \text{ litres}$$

Solution 25 (i)

Correct option: (c) ₹1000

r = 8%, I = ₹2000, n = 24

$$I=P\times\frac{n(n+1)}{2}\times\frac{r}{1200}$$

$$\therefore 2000=P\times\frac{24(25)}{2}\times\frac{8}{1200}$$

$$\therefore P=\text{Rs.}1000$$

Solution 25 (ii)

Correct option: (b) ₹24000

Total amount deposited = 1000×24 = ₹24000

Solution 25 (iii)

Correct option: (c) ₹26000

Maturity amount = Total amount deposited + Interest = ₹26000

Solution 25 (iv)

Correct option: (c) 12

r = 8%, I = ₹52, P = ₹100

$$I = P \times \frac{n(n+1)}{2} \times \frac{r}{1200}$$

$$\therefore 52 = 100 \times \frac{n(n+1)}{2} \times \frac{8}{1200}$$

$$\therefore n = 12$$

ICSE Board
Class X Mathematics
Board Paper
Semester 2 - 2022

Time: 90 minutes
Marks : 40

Maximum Marks: 40
*Time allowed: **One and a half hours***

Answers to this Paper must be written on the paper provided separately.

*You will not be allowed to write during the first **10 minutes**.*
This time is to be spent reading the question paper.

The time given at the head of this Paper is the time allowed for writing the answers.

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*Attempt all questions from **Section A** and any three questions from **Section B**.*
*The intended marks for questions or parts of questions are given in **brackets [ ]**.*

SECTION A
(Attempt all questions from this section)

Question 1:

Choose the correct answers to the questions from the given options. (Do not copy the question, Write the correct answer only.)

(i) The probability of getting a number divisible by 3 in throwing a dice is:

(a) $\frac{1}{6}$

(b) $\frac{1}{3}$

(c) $\frac{1}{2}$

(d) $\frac{2}{3}$

Ans.

(b) $\frac{1}{3}$
Sample space of dice = 1, 2, 3, 4, 5, 6, = (6)

Favourable out once which is divisible by 3 are -3, 6 = (2)

$$P(E) = \frac{Favourable\ out\ cone}{total\ out\ cone} = \frac{2}{6} = \frac{1}{3}$$

(ii) The volume of a conical tent is $462\ m^3$ and the area of the base is $154\ m^2$, the height of the cone is:

(a) $15\ m$
(b) $12\ m$
(c) $9\ m$
(d) $24\ m$

Ans. (c) 9m

Ans. volume of tent = $462\ m^3$
Area of base $= 154\ m^2$
Height =?
Rol of cone $= \frac{1}{3}\pi r^2 h$

$$462 = \frac{1}{3} \times \left[\frac{22}{7} \times r^2\right] \times h$$
$$462 = \frac{1}{3} \times [154] \times h$$
$$\frac{462 \times 3}{154} = h$$
$$9m\ = h$$

(iii) The median class for the given distribution is:

Class Interval	0 - 10	10 - 20	20 - 30	30 - 40
Frequency	2	4	3	5

(a) 0 - 10
(b) 10 - 20
(c) 20 - 30
(d) 30 - 40

Ans. (c)20 - 30

Ans.

Class intorvel	Frequency	C.F
0 - 10	2	2
10 - 20	4	6
20 - 30	3	9
30 - 40	5	14

$n = 14$ which is even so

Median $= \frac{n}{2} = \frac{14}{2}$

= 7th term therefore 20 - 30 is median class

(iv) If two lines are perpendicular to one another then the relation between their slopes m_1 and m_2 is:

(a) $m_1 = m_2$

(b) $m_1 = \frac{1}{m_2}$

(c) $m_1 = -m_2$

(d) $m_1 \times m_2 = -1$

Ans. (d) $m_1 \times m_2 = -1$

Ans. If perpendicular then slope m_1 and slope m_2 are related to $m_1 \times m_2 = -1$

(v) A lighthouse is 80 m high. The angle of elevation of its top from a point 80 m away from its foot along the same horizontal line is:

(a) 60°

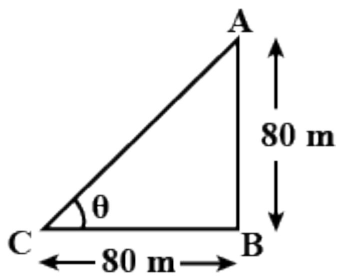
(b) 45°

(c) 30°

(d) 90°

Ans. (b) 45°

Ans.



Height of the lighthouse = 80 m

Angle of elevation = ?

Dis from foot = 80 m

Let AB is height of height house and BC is distance from foot of light house

$$\Rightarrow \ tan\,\theta = \frac{Perpendicular}{Base}$$

$$\Rightarrow tan\,\theta = \frac{80}{80} = 1$$

$$\Rightarrow tan\,45° = 1$$

Therefore, $\theta = 45°$

(vi) The modal class of a given distribution always corresponds to the:

(a) interval with highest frequency
(b) Interval with lowest frequency
(c) The first interval
(d) The last interval

Ans. (a) interval with highest frequency

Ans. We know that modal class of any distribution is height frequency
Therefore = interval with high frequency

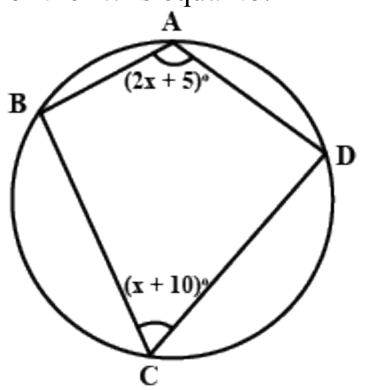
(vii) The coordinates of the point P(-3, 5) on reflecting on the x-axis are:

(a) (3,5)
(b) $(-3,-5)$
(c) $(3,-5)$
(d) $(-3,5)$

Ans. (b) $(-3,-5)$

Ans. On reflecting x axis value of y (ordinate) change
Therefore $P(-3, 5) \xrightarrow{\text{x axis}} (-3, -5)$

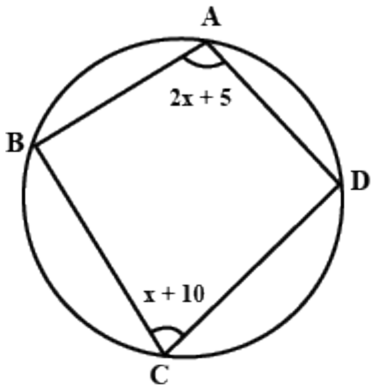
(viii) ABCD is a cyclic quadrilateral. If $\angle BAD = (2x + 5)°$ and $\angle BCD = (x + 10)°$ or then x is equal to:



(a) 65°
(b) 45°
(c) 55°
(d) 5°

Answer : (c) 55°

Ans.



According to law of cyclic properties that sum of opposite angle is 180°

Hence $\angle A + \angle C = 180$

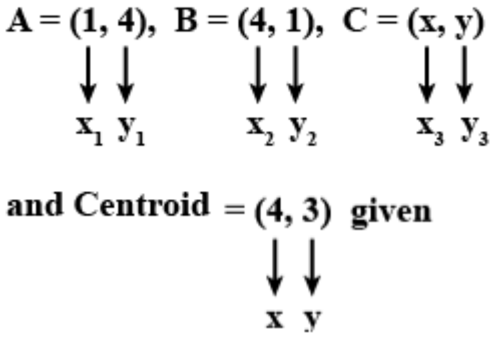
$$\Rightarrow 2x + 5 + x = 10 = 180$$
$$\Rightarrow 3x + 15 = 180$$
$$\Rightarrow 3x = 180 - 15$$
$$\Rightarrow 3x = 165$$
$$\Rightarrow x = \frac{165}{3}$$
$$\Rightarrow x = 55°$$

(ix) A (1,4), B (4,1) and C $(x, 4)$ are the vertices of ΔABC. If the centroid of the triangle is G (4,3) then x is equal to:

(a) 2
(b) 1
(c) 7
(d) 4

Ans. (c) 7

Ans.



Centroid (G) $x = \frac{x_1+x_2+x_3}{3}$ and $y = \frac{y_1+y_2+y_3}{3}$

$\Rightarrow 4 = \frac{1+4+x}{3}$ and $3 = \frac{4+1+4}{3}$

$\Rightarrow 12 = 5 + x$ and $9 = 9$

$$\Rightarrow 12 - 5 = x$$
$$\Rightarrow 7 = x$$

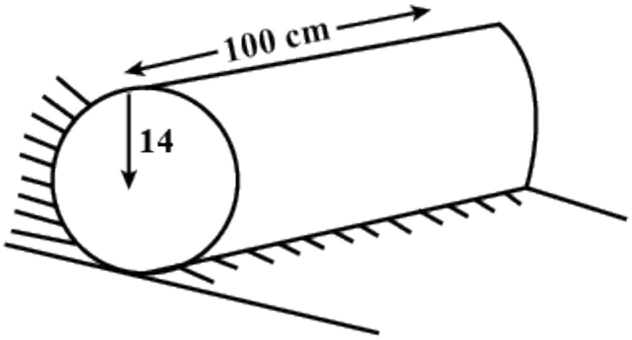
(x) The radius of a roller 100 cm long is 14 cm. The curved surface area of the roller is

(Take $\pi = \frac{22}{7}$)

(a) $13200\ cm^2$
(b) $15400\ cm^2$
(c) $4400\ cm^2$
(d) $8800\ cm^2$

Ans. (d) $8800\ cm^2$

Ans.



radius = 14 cm
long / height = 100 cm
Attention - Read the question very carefully to understand which value is radius and which is height.

$$C\,S\,A\ = 2\pi\, r\, h$$
$$= 2 \times \frac{22}{7} \times 14 \times 100$$
$$= 8800\ cm^2$$

SECTION B
(Attempt all questions from this section)

Question 2:

(i) Prove that:

$$\frac{1}{1 + sin\theta} + \frac{1}{1 - sin\,\theta} = 2sec^2\theta$$

Ans.

$$LHS = \frac{1}{1 + sin\,\theta} + \frac{1}{1 - sin\,\theta}$$
$$= \frac{1 - sin\,\theta + 1 + 1\,sin\,\theta}{(1 + sin\,\theta)\,(1 - sin\,\theta}$$
$$= \frac{1 - sin\,\theta + 1 + sin\,\theta}{(1 + sin\,\theta(1 - sin\,\theta)}$$
$$= \frac{2}{1 - sin^2\theta}$$

$$= \frac{2}{cos^2 x}$$
$$= 2 \times \frac{1}{cos^2 x}$$
$$= 2\,sec^2\,x$$

(i) L. H. S

$$\frac{1}{1 + sin\,\theta} + \frac{1}{1 - sin\,\theta}$$
$$\Rightarrow \frac{(1 - sin\,\theta) + (1 + sin\,\theta)}{(1 + sin\,\theta)\,(1 - sin\,\theta)}$$
$$\Rightarrow \frac{1 - sin\,\theta + 1 + sin\,\theta}{1 - sin^2\theta}$$
$$\Rightarrow \frac{2}{cos\,\theta^2}\ [1 - sin^2\theta = cos^2\theta$$
$$\Rightarrow 2\,sec^2\theta\ [\frac{1}{cos} = sec]$$
$$\Rightarrow R.H.S$$

(ii) Find a if A $(2a + 2, 3)$, B $(7,4)$ and C $(2a + 5,2)$ are collinear.

Ans.

If any 3 point collinear (straight)
Then slope of first 2 point = slope of Last 2 point
Slope of AB = slope of BC

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{y_2 - y_1}{x_2 - x_1}$$

⇒ first we find slope of AB

$$\textbf{A} = (\underset{x_1}{\underset{\downarrow}{2a+2}},\ \underset{y_1}{\underset{\downarrow}{3}})\quad \textbf{and}\quad \textbf{B} = (\underset{x_2}{\underset{\downarrow}{7}},\ \underset{y_2}{\underset{\downarrow}{4}})$$

Slope of $AB = \frac{y_2 - y_1}{x_2 - x_1} =$
$\frac{4-3}{7-(2a+2)}$ $[do\ not\ forget\ put\ bracket\ during\ putting\ value\ of\ x_1]$

$$\Rightarrow \frac{1}{7 - 2a - 2}$$
$$\Rightarrow \frac{1}{5 - 2a}$$

Now slope of BC

$$\textbf{B} = (\underset{x_1}{\underset{\downarrow}{7}},\ \underset{y_1}{\underset{\downarrow}{4}})\quad \textbf{and}\quad \textbf{C} = (\underset{x_2}{\underset{\downarrow}{2a+5}},\ \underset{y_2}{\underset{\downarrow}{2}})$$

Slope $BC = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2-4}{2a+5-7}$

$$\Rightarrow \frac{-2}{2a-2} = \frac{-2}{2(a-1)}$$
$$\Rightarrow \frac{-1}{(a-1)}$$

But slope AB and BC equal due to collinear point

So $\frac{1}{5-2\,a} = \frac{-1}{a-1}$

$$\Rightarrow a - 1 = -5 + 2a$$
$$\Rightarrow -1 + 5 = 2a - a$$
$$\Rightarrow 4 = a$$

Attention [do not use area of Δ formula for above it is CBSE method]

(iii) Calculate the mean of the following frequency distribution.

Class Interval	5 - 15	15 - 25	25 - 35	35 - 45	45 - 55
Frequency	2	6	4	8	4

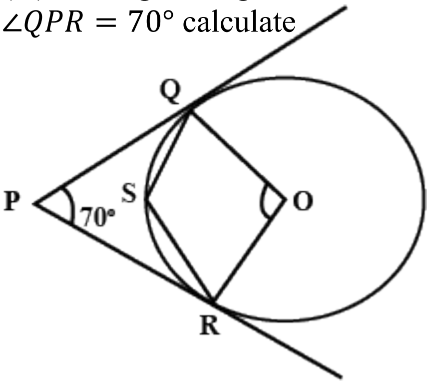
Ans.

Class interval	Frequency(F)	Class mark (x)	fx
5 - 15	2	10	20
15 - 25	6	20	120
25 - 35	4	30	120
35 - 45	8	40	320
45 - 55	4	50	200

$\sum f = 24 \quad \sum fx = 780$

mean $\frac{\varepsilon\, x.f}{\varepsilon\, f} = \frac{780}{24} = 32.5$

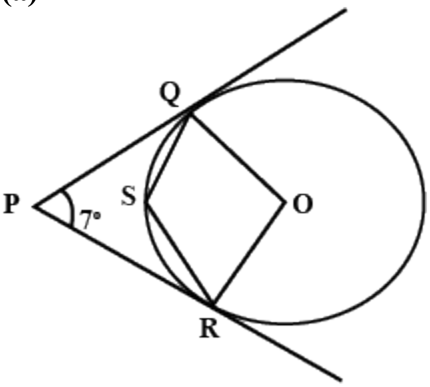
(iv) In the given figure O is the center of the circle. PQ and PR are tangent and $\angle QPR = 70°$ calculate



(a) $\angle QOR$
(b) $\angle QSR$

Ans.

(a)

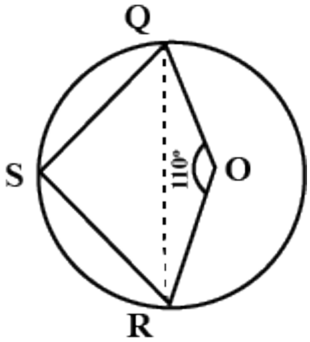


$$\angle Q = 90°$$

$\angle R = 90°$ [straight line drawn from centre to tangent is 90°]
$\angle O + 90° + 90° + 70° = 360°$ [sum of all angles of a quadrilateral is 360°]

$$\angle QOR + 250° = 360°$$
$$\angle QOR = 360° - 250°$$
$$= 110°$$

(b)



from port (a)

$$\angle QSR = ?$$

Draw a line QR which is chord.

$$\angle QSR = \frac{1}{2}[\angle QOR]$$
$$\angle QSR = \frac{1}{2} \times 110°$$
$$\angle QSR = 55°$$

[angle substend by a chord at centre is double then any other point on circle by same chord]

Question 3:

(i) A bag contains 5 white, 2 red and 3 black balls. A ball is drawn at random. What is the probability that the ball drawn is a red ball?

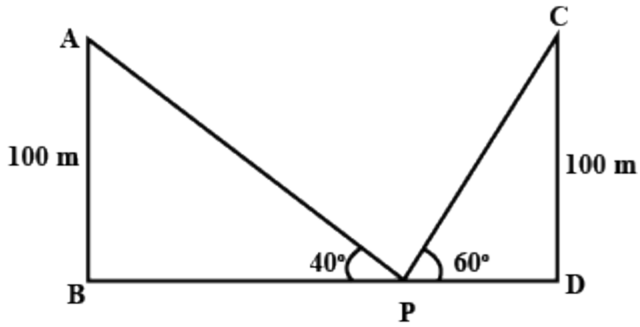
Ans. white ball = 5
Red ball = 2
Black ball = 3
Total ball = W = R + B
= 5 + 2 + 3
= 10
Favourable outcome = red ball
= 2
P(Event) $= \frac{Fourable\ outcone}{total\ outcone} = \frac{2}{10} = \frac{1}{5}$

(ii) A Solid cone of radius 5 cm and height 9 cm is melted and made into small cylinders of radius of 0.5 cm and height 1.5 cm. Find the number of cylinders so formed.

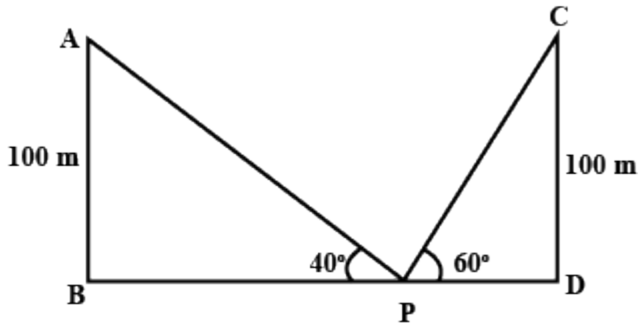
Ans. radius of cone = 5 cm
Hight = 9 cm
First find vol of cone = ?
Vol of cone $= \frac{1}{3}\pi\, r^2 h = \frac{1}{3} \times \frac{22}{7} \times 5^2 \times 9 = \pi \times 25 \times 3 = 75\pi\ cm^2$
Now radius of cylinder = 0.5 cm
Height, h = 1.5 cm
Vol of 1 cylinder $\pi\, r^2 h = \pi \times 0.5^2 \times 1.5 = \pi \times 0.25 \times 1.5 = \pi \times cm^3$
Let no of cylinder = n
According law of conservation of mass vol of all cylinder formed = vol of 1 cone method

$$n \times 0.375\,\pi = 75\pi$$
$$n = \frac{75}{0.375}$$
$$= 200$$

(iii) Two lamp posts AB and CD each of height 100 m are on either side of the road. P is a point on the road between the two lamp posts. The angles of elevation of the top of the lamp posts from the point P are 60° and 40°. Find the distances PB and PD.



Ans.



In $\Delta\ ABP$

Tan $40 = \frac{AB}{BP}$

$$0.84 = \frac{100}{BP}$$

$$BP = \frac{100}{0.84} = 119\ m$$

So $PB = 119\ m$

$$PD = \frac{100\sqrt{3}}{3}m$$

In $\Delta\ CPD$

$$tan\ 60 = \frac{CD}{PD}$$

$$\sqrt{3} = \frac{100}{PD}$$

$$PD = \frac{100}{\sqrt{3}}$$

$$PD = \frac{100 \times \sqrt{3}}{\sqrt{3} \times \sqrt{3}}$$

$$PD = \frac{100\sqrt{3}}{3}$$

$$PD = \frac{100 \times 1.732}{3}$$

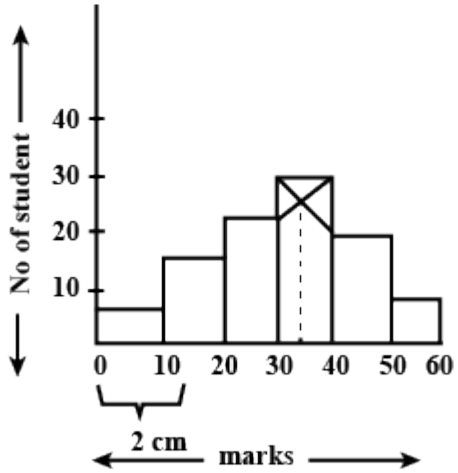
$$= \frac{173.2}{3}$$

$$= 57.73$$

(iv) Marks obtained by 100 students in an examination are given below.

Marks	0 - 10	10 - 20	20 -3 0	30 - 40	40 - 50	50 - 60
No. of students	5	15	20	28	20	12

Draw a histogram for the given data using a graph and find the mode.
Take 2 cm = 10 marks along one axis and 2 cm = 10 students along the other axis.
Ans.



mode = 34 (Approx)

Question 4:

(i) Find a point P which divides internally the line segment joining the points A (–3,9) and B (1,– 3) in the ratio 1:3.

(ii) A letter of the word SECOŃDARY is selected at random. What is the probability that the letter selected is not a vowel?

(iii) Use a graph paper for this question. Take 2cm- 1 unit along both the axes.

(a) Plot the points A(0,4), B(2,2), C(5,2) and D(4,0). E (0,0) is the origin.

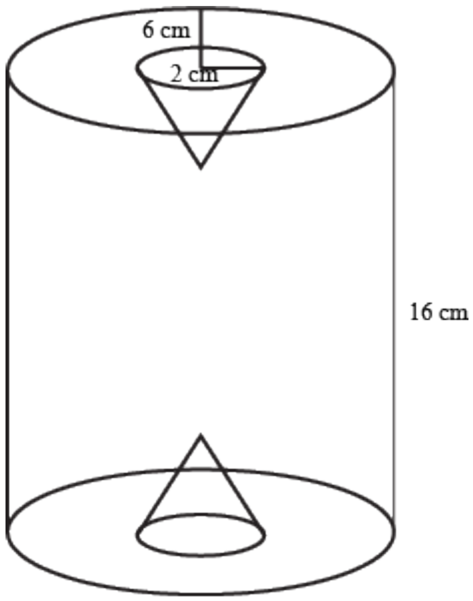
(b) Reflect B, C, D on the y-axis and name then as B’, C’ and D’ respectively.

(c) Join the points ABCDD’C’B’ and A in order and give a geometrical name to the closed figure

(iv) A solid wooden cylinder is of radius 6 cm and height 16 cm. Two cones each of radius 2 cm and height 6 cm are

drilled out of the cylinder. Find the volume of the remaining solid.

Take $\pi = 22/7$



Ans.

[i]

$$\begin{array}{ccccc} & m & & n & \\ & \uparrow & & \uparrow & \\ A \leftarrow & 1 & \rightarrow P \leftarrow & 3 & \rightarrow B \\ (-3,9) & & (x,y) & & (1,-3) \\ \downarrow\ \downarrow & & & & \downarrow\ \downarrow \\ x_1\ y_1 & & & & x_2\ y_2 \end{array}$$

Let the co-ordinate of P(x, y)

$$x = \frac{mx_2 + nx_1}{m+n}$$

$x = \frac{1 \times 1 + 3(-3)}{1+3}$

$$x = \frac{1 - g}{4}$$

$$x = \frac{-8}{4}$$

$$x = -2$$

$$y = \frac{my_2 + ny_1}{m+n}$$

$y = \frac{1 \times (-3) + 3 \times g}{1+3}$

$$y = \frac{-3 + 2}{4}$$

$$y = \frac{24}{4}$$

$$y = 6$$

Therefore coordinate of $p(x, y)$ = $p(-2,6)$

[ii]

Total outcome = S, E, C, O, N, D, A, R, Y

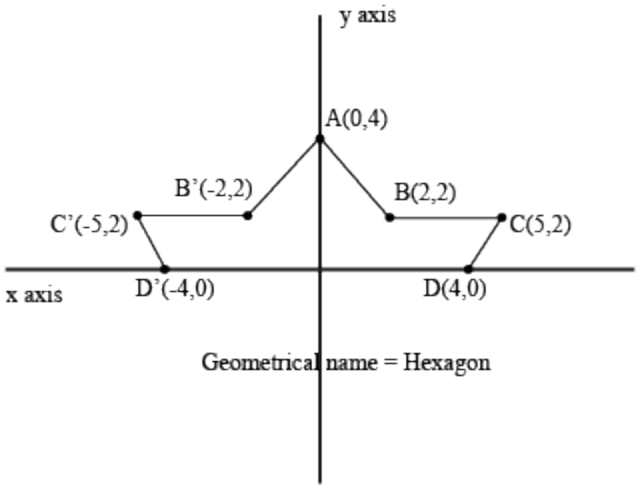
= 9

Favorable Outcome = S, C, N, D, R, Y
= 6

P(E) = $\frac{Favorable\ Outcome}{Total\ Outcome}$

= $\frac{6}{9} = \frac{2}{3}$

[iii]



[iv]

Radius of cylinder = 6 cm
Height = 16 cm
Volume of cylinder = $\pi r^2\ h$

$= \pi \times 6^2 \times\ 16$

= 36 x 16π

Radius of each cone = 2 cm

Height = 6 cm

Vol of = $\frac{1}{3} \times\ \pi\ \times 2^2 \times\ 6$

Vol of 2 cone = 8π

Vol of 2 cone = 2 × vol of 1 cone

Vol of 2 cone = 2 × 8π

Vol of 2 cone = 16π

Vol of remaining = vol of cylinder − vol of 2 cones solved

$= 36 \times 16\pi - 16\pi$

$= 16\pi(36 - 1)$

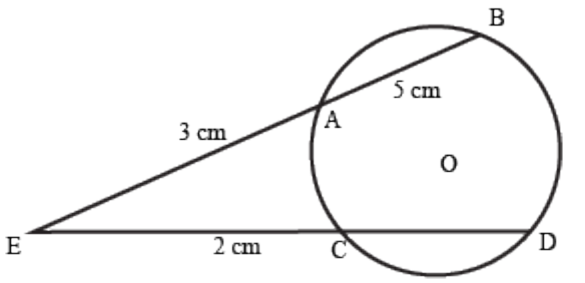
$= 16\pi \times 35$

$= 16 \times 22 \times 5$

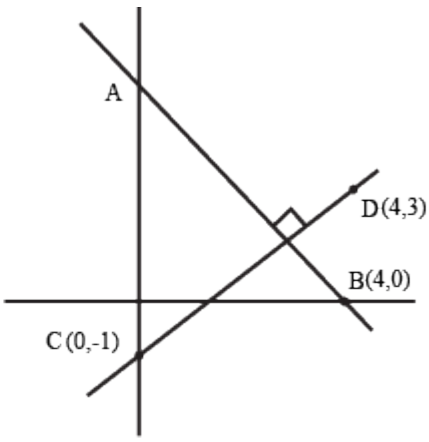
$= 1760\ cm^3$

Question 5:

(i) Two chords AB and CD of a circle intersect externally at E. If EC = 2 cm, EA = 3 and AB = 5 cm cm, Find the length of CD.



(ii) Line AB is perpendicular to CD. Coordinate of B, C and D are respectively (4,0), $(0, -1)$ and (4,3).



Find:

(a) Slope of CD

(b) Equation of AB

(iii) Prove that :

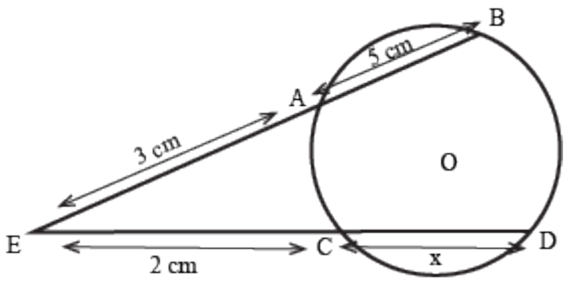
$$(1 + sin\ \theta)^2\ +\ (1 - sin\theta)^2/2cos^2\theta = sec^2\theta\ + tan^2\theta$$

(iv) The name of the following distribution is 50. Find the unknown frequency.

Class interval	Frequency
0 - 20	6
20 - 40	f
40 - 60	8
60 - 80	12
80 - 100	8

Ans.

[i]



If the two lines from external point intersect the circle at two points respectively, then:

$$EA \times EB\ = EC \times ED$$

$$\Rightarrow\ 3\ \times\ (3 + 5)\ =\ 2\ \times\ (2 + x)$$

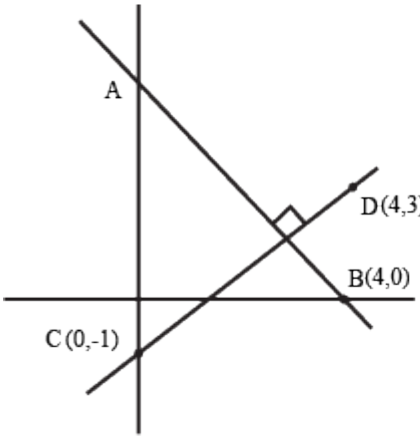
$$\Rightarrow g + 15 = 4 \times 2x$$

$$\Rightarrow 24 - 4\ =\ 2x$$

$$\Rightarrow \frac{20}{2} = x$$

$$\Rightarrow 10 = x$$

[ii]



Slope CD = ?

C = (0, −1), D = (4, 3)

$\downarrow \downarrow \qquad \downarrow \downarrow$

$\boldsymbol{x_1}\ \boldsymbol{y_1} \qquad \boldsymbol{x_2}\ \boldsymbol{y_2}$

$$\boldsymbol{Slope\ CD\ = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3-(-1)}{4-0} = \frac{3+1}{4} = \frac{4}{4} = 1}$$

$\boldsymbol{Slope\ CD\ = 1}$

[b] Let slope of AB is m_2

According to law of perpendicularity

$m_1 \times m_2 = -1$

$\Rightarrow 1 \times m_2 = -1 \qquad [m_1 = 1\, from\ part\ a]$

$$m_2 = \frac{-1}{1} = -1$$

Now equation AB = ?

B = (4,0)

$x_1\, y_1$

equation of a line of one coordinate and slope given is

$$y - y_1 = m(x - x_1)$$

$$\boldsymbol{y - 0 = -1(x - 4)}$$

$\boldsymbol{y = -x + 4}$ $\quad$ $\boldsymbol{[m = slope\ of\ AB = m_2]}$

$$\boldsymbol{x + y = 4}$$

$$\boldsymbol{x + y - 4 = 0}$$

[iii]

L.H.S

$$\boldsymbol{\frac{(1 + sin\theta)^2 + (1 - sin\theta)^2}{2\ cos^2\theta}}$$

$$\boldsymbol{= \frac{1 + sin^2\theta + 2sin\theta + 1 + sin^2\theta - 2sin\theta}{2\ cos^2\theta}}$$

$$\boldsymbol{= \frac{2 + 2\ sin^2\theta}{2\ cos^2\theta}}$$

$$\boldsymbol{= \frac{2\ (1 + sin^2\theta)}{2\ cos^2\theta}}$$

$$\boldsymbol{= \frac{1 + sin^2\theta}{cos^2\theta}}$$

$\boldsymbol{= \frac{1}{cos^2\theta} + \frac{sin^2\theta}{cos\ \ \theta}}$ $\quad$ $\boldsymbol{[\ \frac{1}{cos\ \ \theta} = sec^2\theta\ , \frac{sin\theta}{cos\theta} = tan\theta]}$

$$\boldsymbol{= sec^2\theta + tan^2\theta}$$

$$\boldsymbol{= R.H.S}$$

[iv]

Class interval	Classwork	Frequency	fx
0 - 20	10	6	60
20 - 40	30	f	30f
40 - 60	50	8	400
60 - 80	70	12	840
80 - 100	90	8	720

$$\sum f = 34 + f \qquad \sum f \,.\, f = 2020 + 30f$$

Mean = 50 (given)

mean = $\frac{\Sigma fx}{\Sigma f}$

50 = $\frac{2020+30f}{34+f}$

$$50 \times (34 + f) = 2020 + 30f$$

$$1700 + 50f = 2020 + 30f$$

$$50f - 30f = 2020 - 1700$$

$$20f = 320$$

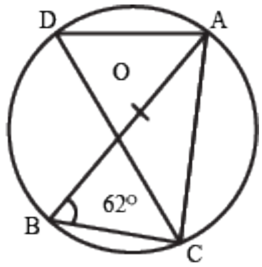
$$f = \frac{320}{20}$$

Question 6 :

(i) Prove that:

$$1 + tan^2\theta/(1 + sec\theta) = sec\theta$$

(ii) In the given figure A, B, and D are points on the circle with centre O.



Given, ∠ABC = 62°.

Find:

(a) ∠ADC

(b) ∠CAB

(iii) Find the equation of a line parallel to the line $2x + y - 7 = 0$ and passing through the intersection of the line $x + y - 4 = 0$ and $2x - y = 8$.

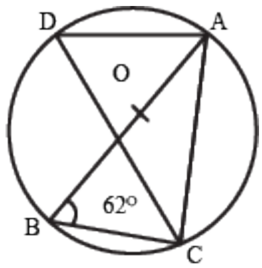
(iv) Marks obtained by students in an examination are given below.

Marks	10 - 20	20 - 30	30 - 40	40 - 50	50 - 60	60 - 70
No. of Student	3	8	14	9	4	2

Using graph paper, draw an ogive and estimate the median marks. Take 2 cm = 10 marks along one axis and 2 cm = 5 students along the other axis.

Ans.

[i]



L.H.S = $\mathbf{1 + \frac{tan^2\theta}{1+sec\theta}}$

$$= \frac{1 + sec\theta \ + \ tan^2\theta}{1 + sec\theta}$$

$= \frac{1+sec\theta + sec^2\theta - 1}{1+sec\theta}$ $\quad [tan^2\theta = sec^2\theta - 1]$

$$= \frac{sec\theta \ + \ sec^2\theta}{1 + sec\theta}$$

$$= \frac{sec\theta \ + (1 + \ sec\theta)}{(1 + \ sec\theta)}$$

$$= sec\theta \ = \ R.H.S$$

[ii]

[a] ∠ADC = ?

∠ADC = ∠ABC

∠ADC = 62° [same chord subtend equal angles on the circle]

[b] ∠CAB = ?

∠ACB = 90° [angle on semi circle = 90°]

Now in ΔABC

∠ACB + ∠CBA + ∠BAC = 180°

90° + 62° + ∠ACB = 180°

152° + ∠ACB = 180°

∠ACB = 180° – 152°

∠ACB = 28°

[iii]

The equation $\boldsymbol{2x + y - 7 = 0}$ can be written as $y = 7 - 2x$

Comparing the above equation with $y = mx + c$, slope of the equation $\boldsymbol{2x + y - 7 = 0}$ will be:

Slope $= m = -2$

Let $m = m_1$ and a line parallel to any line has slope m_2 then,

$$m_1 = m_2 = -2$$

Therefore, slope of line parallel to $\mathbf{2x + y - 7 = 0}$ is -2

Now, it is given that $\boldsymbol{x + y - 4 = 0}$ and $\mathbf{2x - y = 8}$ intersects, so

$$\boldsymbol{x + y - 4 = 0}$$

$$\mathbf{2x - y = 8}$$

—-------------

$$\mathbf{3x = 12}$$

$$\boldsymbol{x = 4}$$

Therefore, the parallel line cut at (4,0)

Putting value of $x = 4$ in the equation of line, we get:

$$\boldsymbol{x + y - 4 = 0}$$

$$\mathbf{4 + y = 4}$$

$$\boldsymbol{y = 4 - 4}$$

$$\boldsymbol{y = 0}$$

So now the equation of the line parallel to $2x + y - 7 = 0$ and passing through the intersection of the line $x + y - 4 = 0$ and $2x - y = 8$ will be:

$$\boldsymbol{y - y_1 = m\,(x - x_1)}$$

$$\boldsymbol{y - 0 = -2\,(x - 4)}$$

$$\boldsymbol{y = -2x + 8}$$

$$\boldsymbol{y + 2x - 8 = 0}$$

[iv]

Marks	No of student	cf
10 - 20	3	3
20 - 30	8	11
30 - 40	14	25
40 - 50	9	34
50 - 60	4	38
60 - 70	2	40

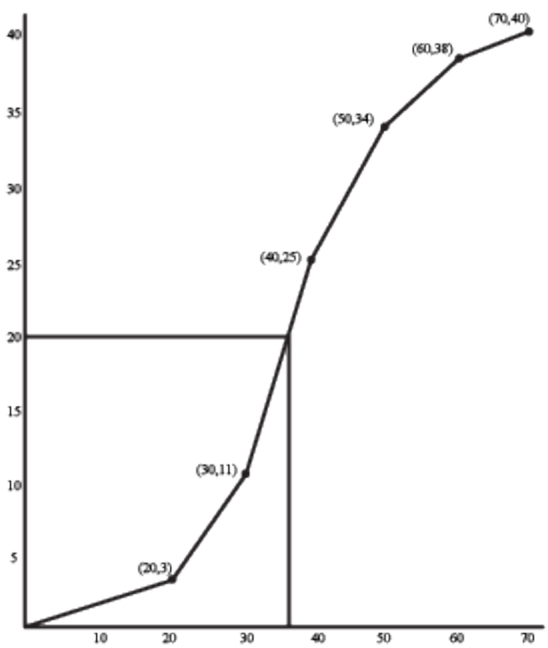
Now we will plot the points $(20,3)$, $(30,11)$, $(40,25)$, $(50,34)$, $(60,38)$ and $(70,40)$ on graph

To get the median $n = 40$, which is even:

Median = $\frac{n}{2} = \frac{40}{2} = 20$

Take a point 20 on y axis and through it draw a line parallel to x axis which meet current at A. Through A draw a perpendicular on x axis which meet on 3 > 5

So median is = 3 > 5



ICSE X Mathematics
2022-23

Maximum Marks: 80
Time allowed: Two and half hours
Answers to this Paper must be written on the paper provided separately.
You will not be allowed to write during first 15 minutes.
This time is to be spent in reading the question paper.
The time given at the head of this Paper is the time allowed for writing the answers.

Attempt all questions from Section **A** and any four questions from Section **B**.
All working, including rough work, must be clearly shown, and must be done on the same sheet as the rest of the answer.
Omission of essential working will result in loss of marks.
The intended marks for questions or parts of questions are given in brackets [ ]
Mathematical tables and graph papers are provided.

SECTION A (40 Marks)

(Attempt **all** questions from this **Section**.)

Question 1

Choose the correct answers to the questions from the given options. [15]
(Do not copy the questions, write the correct answers only.)

(i) If $\begin{bmatrix} 2 & 0 \\ 0 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 \\ -8 \end{bmatrix}$, the value of x and y respectively are:

(a) $1, -2$
(b) $-2, 1$
(c) 1, 2
(d) $-2, -1$

Answer: (a)

Explanation:

$2x + 0y = 2$
$x = 1$
$0x + 4y = -8$
$y = -2$

(ii) If x – 2 is a factor of $x^3 - kx - 12$, then the value of k is:
(a) 3
(b) 2
(c) -2
(d) -3
Answer: (c)
Explanation:

$x - 2 = 0$

$x = 2$

Substituting x in the equation,

$2^3 - 2k - 12 = 0$

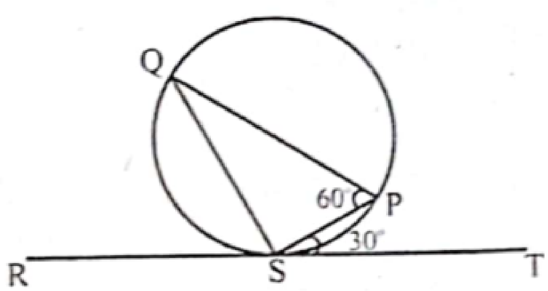
$8 - 2k - 12 = 0$

$-2k - 4 = 0$

$-2k = 4$

$k = -2$

(iii) In the given diagram RT is a tangent touching the circle at S. If $\angle PST = 30^\circ$ and $\angle SPQ = 60^\circ$ then $\angle PSQ$ is equal to:



(a) 40°
(b) 30°
(c) 60°
(d) 90°
Answer: (d)
Explanation:

Given, $\angle PST = 30^\circ$ and $\angle SPQ = 60^\circ$

From alternate segment theorem:

$\angle PQS = \angle PST = 30^{\circ}$ ...(i)

In ΔPQS,

∠PQS + ∠PSQ + ∠SPQ = 180°

$30^{\circ} + \angle PSQ + 60° = 180°$ (from (i))

∠PSQ = $180° - 30° - 60°$

∠PSQ = $90°$

(iv) A letter is chosen at random from all the letters of the English alphabets. The probability that the letter chosen is a vowel is:

(a) $\frac{4}{26}$

(b) $\frac{5}{26}$

(c) $\frac{21}{26}$

(d) $\frac{5}{24}$

Answer: (b)

Explanation:

The number of English alphabets = 26

The number of vowels in English alphabets = 5

The probability that the letter chosen is a vowel = The number of vowels in English alphabets / The number of English alphabets

The probability that the letter chosen is a vowel = 5/26

(v) If 3 is a root of the quadratic equation $x^2 - px + 3 = 0$ then p is equal to:

(a) 4

(b) 3

(c) 5

(d) 2

Answer: (a)

Explanation:

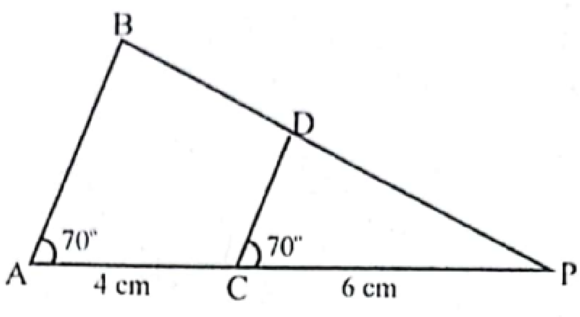
Substitute 3 in the equation,

$$3^2 - p(3) + 3 = 0$$

$$9 - 3p + 3 = 0$$

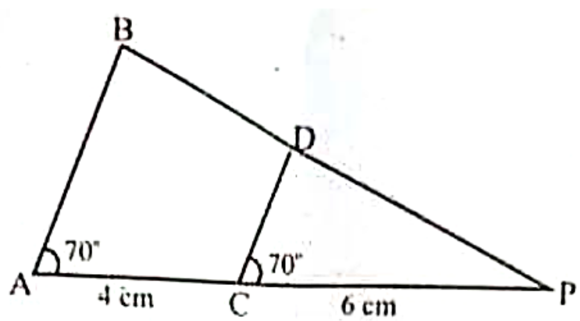
$12 - 3p = 0$
$- 3p =- 12$
$p = 4$

(vi) In the given figure $\angle BAP = \angle DCP = 70^{\circ}, PC = 6\ cm$ and $CA = 4\ cm$, then $PD: DB$ is:



(a) $5: 3$
(b) $3: 5$
(c) $3: 2$
(d) $2: 3$

Answer: (c)
Explanation:



In the given figure,

$\angle BAP = \angle DCP$

$\therefore AB \parallel CD$ {Corresponding angles are equal}

$\Rightarrow \frac{PC}{CA} = \frac{PD}{DB}$ {Applying Basic Proportionality Theorem}

$\Rightarrow \frac{PD}{DB} = \frac{6}{4}$

$\Rightarrow PD: DB = 3: 2$

(vii) The printed price of an article is ₹3080. If the rate of GST is 10% then the GST charged is:
(a) ₹154
(b) ₹308
(c) ₹ 30.80
(d) ₹ ₹15.40
Answer: (b)
Explanation:
Printed price = 3080, rate of GST = 10%
GST charged will be = $\frac{3080\times10}{100} = 308$

(viii) $(1 + sinA)(1 - sinA)$ is equal to:
(a) $cosec^2 A$
(b) $sin^2 A$
(c) $sec^2 A$
(d) $cos^2 A$
Answer: (d)
Explanation:
Given, $(1 + sinA)(1 - sinA)$ which equals to $1 - sin^2 A$
As we have $sin^2\theta + cos^2\theta = 1 \Rightarrow 1 - sin^2\theta = cos^2\theta$
So, from the above identity, $(1 + sinA)(1 - sinA) = cos^2 A$

(ix) The coordinates of the vertices of $\triangle ABC$ are respectively $(-4, -2)$, $(6, 2)$ and $(4, 6)$. The centroid G of $\triangle ABC$ is:
(a) $(2, 2)$
(b) $(2, 3)$
(c) $(3, 3)$
(d) $(0, -1)$
Answer: (a)
Explanation:
Given parameters are,
$(x_1, y_1) = (-4, -2)$

$(x_2, y_2) = (6,2)$
$(x_3, y_3) = (4, 6)$
The centroid formula of a given triangle can be expressed as,

$$C = \left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$$

$$C = \left(\frac{-4+6+4}{3}, \frac{-2+2+6}{3}\right) = \left(\frac{6}{3}, \frac{6}{3}\right)$$

$$C = (2,\ 2)$$

(x) The nth term of an AP is 2n + 5. The 10th term is:
(a) 7
(b) 15
(c) 25
(d) 45
Answer: (c)
Explanation:

$$t_n = 2n + 5$$

$$t_{10} = 2(10) + 5$$

$$t_n = 25$$

(xi) The mean proportional between 4 and 9 is:
(a) 4
(b) 6
(c) 9
(d) 36
Answer: (b)
Explanation:
Mean proportion between two numbers is defined as the square root of the product of the numbers.
i.e. $M = \sqrt{x \times y}$
Let the number $x = 4$ and $y = 9$.
So, Mean proportion between them is

$$M = \sqrt{4\times9} = \sqrt{36} = 6$$

Therefore, The mean proportional between 4 and 9 is 6.

(xii) Which of the following cannot be determined graphically for a grouped frequency distribution?
(a) Median
(b) Mode
(c) Quartiles
(d) Mean
Answer: (d)
Explanation:
Mean is the sum of the values of a subject divided by number of values.
As it is a single value and cannot be compared and represented in different values, therefore, the determination of mean by the graphical method is not possible.
Mean is a perfect method for the determination of the central tendency of a value.
Hence, mean can't be determined graphically.

(xiii) Volume of a cylinder of height $3\ cm$ is 48π. Radius of the cylinder is:
(a) $48\ cm$
(b) $16\ cm$
(c) $4\ cm$
(d) $24\ cm$
Answer: (c)
Explanation:

Let the radius of the base and height of the cylinder be $r\ cm$ and $h\ cm$ respectively. Then, $h = 3\ cm$

Now, Volume $= 48\pi cm^3$

$\pi r^2 h = 48\pi r^2 = \frac{48}{3} = 16r = 4\ cm$

(xiv) Naveen deposits Rs. 800 every month in a recurring deposit account for 6 months. If he receives Rs 4884 at the time of maturity, then the interest he earns is:
(a) ₹ 84
(b) ₹ 42
(c) ₹ 24
(d) ₹ 284
Answer: (a)
Explanation:

Total money deposited in 6 months = 800 x 6 = Rs. 4800
Money received at the time of maturity = Rs 4884
Therefore, the interest he earns = Rs. 4884 – Rs. 4800 = Rs. 84

(xv) The solution set for the inequation $2x + 4 \leq 14,\ x \in W$ is:
(a) $\{1, 2, 3, 4, 5\}$
(b) $\{0, 1, 2, 3, 4, 5\}$
(c) $\{1, 2, 3, 4\}$
(d) $\{0, 1, 2, 3, 4\}$
Answer: (b)
Explanation:

$2x + 4 \leq 14$

$2x \leq 14 - 4$

$2x \leq 10$

$x \leq 5,\ x \in W$

Therefore, the solution set is {0, 1, 2, 3, 4, 5}

Question 2
(i) Find the value of 'a' if $x - a$ is a factor of the polynomial $3x^3 + x^2 - ax - 81$. [4]

Answer: a = 3
Explanation:
Since $(x - a)$ is a factor of the polynomial $p(x) = 3x^3 + x^2 - ax - 81 = 0$, by factor theorem, we have $p(a) = 0$

$\Rightarrow 3(a)^3 - (a)^2 + a(a) - 81 = 0$

$\Rightarrow 3a^3 = 81$

$\Rightarrow a^3 = 27$

$\Rightarrow a = 3$

(ii) Salman deposits ₹ 1000 every month in a recurring deposit account for 2 year If he receives ₹26000 on maturity, find:
(a) the total interest Salman earns.
(b) the rate of interest.

[4]

Answer: Rs 2000, 8%

Explanation:

Salman deposits in 2 years $(P) = 1000 \times 24$

$(P) = Rs\ 24000$

He receives (A) $= Rs\ 26000.$

(a) Total interest (I) $= A - P = 26000 - 24000 = Rs.\ 2000$

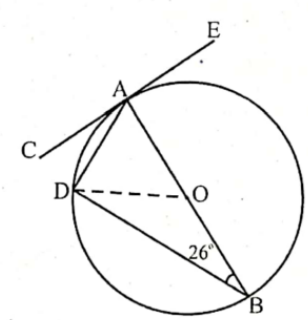
(b) $Interest\ (I) = p \times \frac{(n)(n+1)}{2\times12} \times \frac{r}{100}$

$$2000 = 1000\times\frac{24\times25}{2\times12} \times \frac{r}{100}$$

$$2000 = 250r$$

$$r = 8\%$$

(iii) In the given figure O, is the centre of the circle. CE is a tangent to the circle If ∠ABD=26° , then find:



(a) ∠BDA

(b) ∠BAD

(c) ∠CAD

(d) ∠ODB

[4]

Answer: $\angle BDA = 90^{\circ}, \angle BAD = 64^{\circ}, \angle CAD = 26^{\circ}, \angle ODB = 26^{\circ}$

Explanation:

a) The angle formed by semi-circle is 90°

$$\therefore \angle BDA = 90^{\circ}$$

b) In $\triangle ABD$,

$\angle A + \angle B + \angle D = 180^{\circ}$

$\angle A + 26^{\circ} + 90^{\circ} = 180^{\circ}$

$\angle A = 180^{\circ} - 116^{\circ}$

$\angle A = 64^{\circ}$

$\therefore \angle BAD = 64^{\circ}$

c) The tangent formed by a circle is perpendicular to is diameter.

$\Rightarrow CE \perp AB$

$\therefore \angle CAB = 90^{\circ}$

$\angle CAD + \angle DAB = 90^{\circ}$

$\angle CAD + 64^{\circ} = 90^{\circ}$

$\angle CAD = 26^{\circ}$

d) In $\triangle ODB$,
line OD = line OB [$\because OD$ & OB are radius]
So $\triangle ODB$ is Isosceles $triangle$

$\therefore \angle OBD = \angle ODB$

$\therefore \angle ODB = 26^{\circ}$

Question 3
(i) Solve the following quadratic equation:

$x^2 + 4x - 8 = 0$

Give your answer correct to one decimal place. (Use mathematical tables if necessary.)

[4]

Answer: x = $1.4\ or\ -5.4$
Explanation:
Given quadratic equation is $x^2 + 4x - 8 = 0$

We have a formula for solving quadratic equation $ax^2 + bx + c = 0$ is

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

By using quadratic formula, we have

$$x = \frac{-(4)\pm\sqrt{(4)^2-4(1)(-8)}}{2(1)} = \frac{-4\pm\sqrt{16+32}}{2}$$

$$= \frac{-4\pm\sqrt{48}}{2} = \frac{-4\pm4\sqrt{3}}{2} =- 2\pm2\sqrt{3}$$

$$= 2(-1\pm\sqrt{3}) = 2(-1\pm1.73205) = 2(0.73205)\ or\ 2(-2.73205)$$

$$= 1.4641\ or\ -5.4641$$

$$= 1.4\ or\ -5.4$$

(ii) Prove the following identity: [4]

$$\left(sin^2\theta - 1\right)\left(tan^2\theta + 1\right) + 1 = 0$$

Explanation:

Identities: $sec^2\theta - tan^2\theta = 1$ and $sin^2\theta + cos^2\theta = 1$

$$LHS = \left(sin^2\theta - 1\right)\left(tan^2\theta + 1\right) + 1$$

$=- cos^2\theta \times sec^2\theta + 1$ (From the identity)

$$= (- cos^2\theta \times \frac{1}{cos^2\theta}) + 1$$

$$=- 1 + 1$$

$$= 0$$

$$= RHS$$

Hence, it is proved.

(iii) Use graph sheet to answer this question. Take $2\ cm = 1$ unit along both the axes.

(a) Plot A, B, C where $A(0, 4), B(1, 1)$ and $C(4, 0)$

(b) Reflect A and B on the x-axis and name them as E and D respectively.

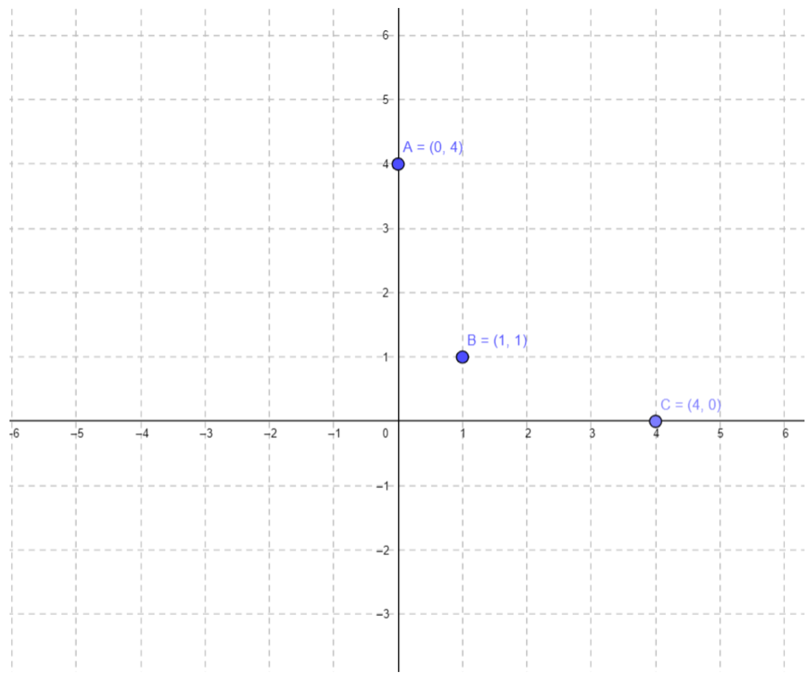
(c) Reflect B through the origin and name it F. Write down the coordinates of F.

(d) Reflect B and C on the y-axis and name them as H and G respectively.

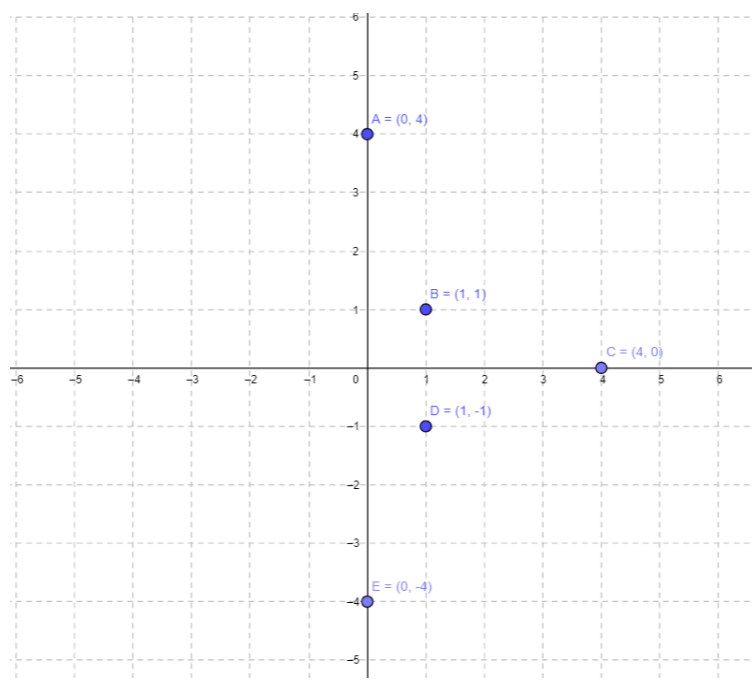
(e) Join points A, B, C, D, E, F, G, H and A in order and name the closed figure formed. [5]

Explanation:

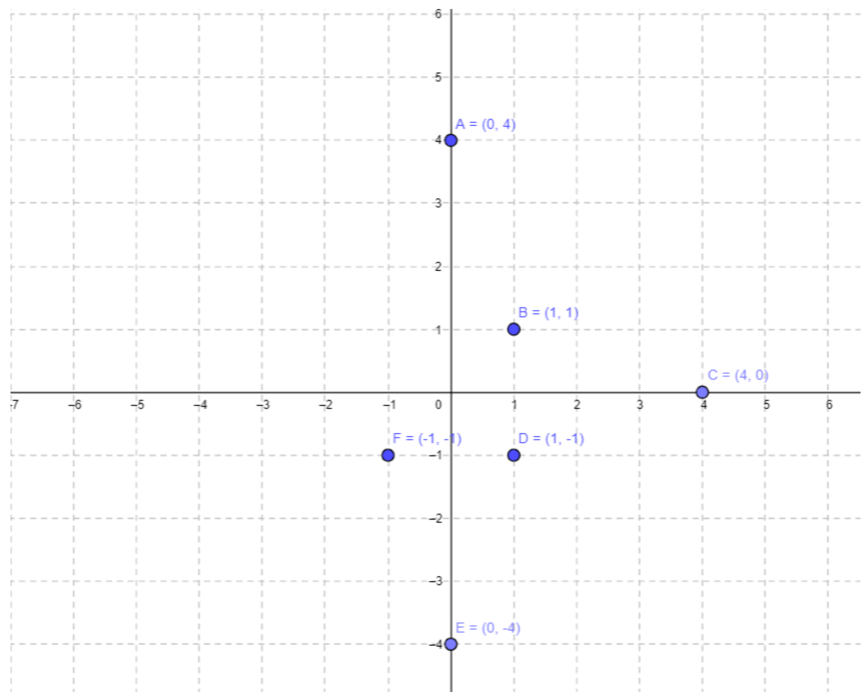
(a) Plot the points A, B, and C using their coordinates. A(0,4) will be located 4 units up the y-axis, B(1,1) will be located 1 unit up the y-axis and 1 unit to the right of the origin, and C(4,0) will be located 4 units to the right of the origin.



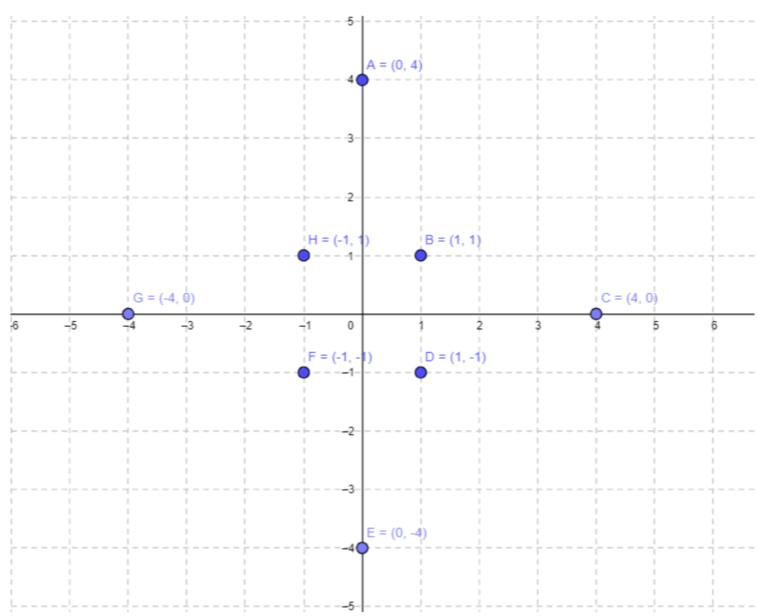
(b) To reflect points A and B on the x-axis, draw a horizontal line passing through A and B. The reflections of A and B on the x-axis will be E and D, respectively. The coordinates of E will be (0,-4), and the coordinates of D will be (1,-1).



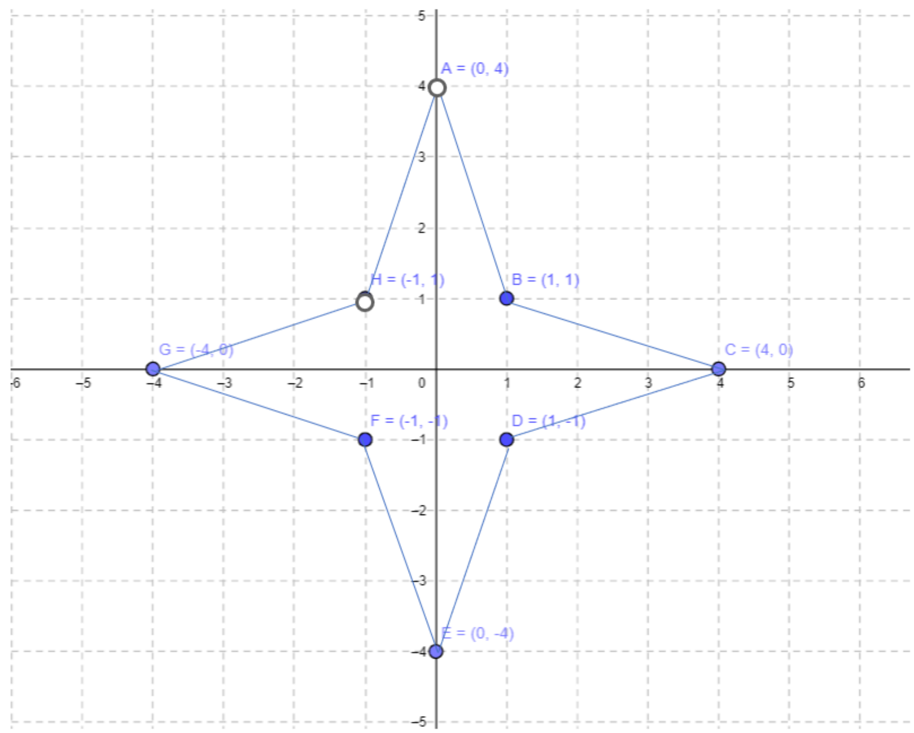
(c) To reflect point B through the origin, draw a line passing through the origin and point B. The reflection of B through the origin will be F. The coordinates of F will be (-1,-1).



(d) To reflect points B and C on the y-axis, draw a vertical line passing through points B and C. The reflections of B and C on the y-axis will be H and G, respectively. The coordinates of H will be (-1,1), and the coordinates of G will be (-4,0).



(e) Join points A, B, C, D, E, F, G, H, and A in order to form a closed figure.
The closed figure formed is a trapezoid.



SECTION B (40 Marks)

(Attempt **any four** questions from this Section.)

Question 4

(i) If A = $\begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}, B = \begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}, C = \begin{bmatrix} 4 & 1 \\ 1 & 5 \end{bmatrix}$ $and\ I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$. $Find\ A(B + C) - 14I$ [3]

Answer: $A(B + C) - 14I$ = $\begin{bmatrix} 0 & 30 \\ 22 & 28 \end{bmatrix}$

Explanation:

B + C = $\begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix} + \begin{bmatrix} 4 & 1 \\ 1 & 5 \end{bmatrix}$

= $\begin{bmatrix} 1+4 & 2+1 \\ 2+1 & 4+5 \end{bmatrix}$

= $\begin{bmatrix} 5 & 3 \\ 3 & 9 \end{bmatrix}$

A(B + C) = $\begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix} \begin{bmatrix} 5 & 3 \\ 3 & 9 \end{bmatrix}$

= $\begin{bmatrix} (1\times 5)+(3\times 3) & (1\times 3)+(3\times 9) \\ (2\times 5)+(4\times 3) & (2\times 3)+(4\times 9) \end{bmatrix}$

= $\begin{bmatrix} 5+9 & 3+27 \\ 10+12 & 6+36 \end{bmatrix}$

= $\begin{bmatrix} 14 & 30 \\ 22 & 42 \end{bmatrix}$

$14\mathrm{I} = 14 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

= $\begin{bmatrix} 14 & 0 \\ 0 & 14 \end{bmatrix}$

Therefore, $A(B+C) - 14I = \begin{bmatrix} 14 & 30 \\ 22 & 42 \end{bmatrix} - \begin{bmatrix} 14 & 0 \\ 0 & 14 \end{bmatrix}$

= $\begin{bmatrix} 14-14 & 30-0 \\ 22-0 & 42-14 \end{bmatrix}$

= $\begin{bmatrix} 0 & 30 \\ 22 & 28 \end{bmatrix}$

(ii) ABC is a triangle whose vertices are A(1, -1), B(0, 4) and C(-6, 4). D is the midpoint of BC. Find the
a) Coordinates of D and
b) Equation of the median AD.

[3]

Answer: a) D is (-3, 4) **b)** The equation of line AD is $5x + 4y = 1$
Explanation:
a) Midpoint of BC = $\left(\frac{0-6}{2}, \frac{4+4}{2}\right)$
= (-3, 4)
Therefore, the point D is (-3, 4).

b) Equation of the median AD:
Equation of the line joining two points

$\frac{y-y_1}{y_2-y_1} = \frac{x-x_1}{x_2-x_1}$

A (1, -1) and D (-3, 4)

$\frac{y-(-1)}{4-(-1)} = \frac{x-1}{-3-1}$

$\frac{y+1}{5} = \frac{x-1}{-4}$

$(-4)(y+1) = 5(x-1)$

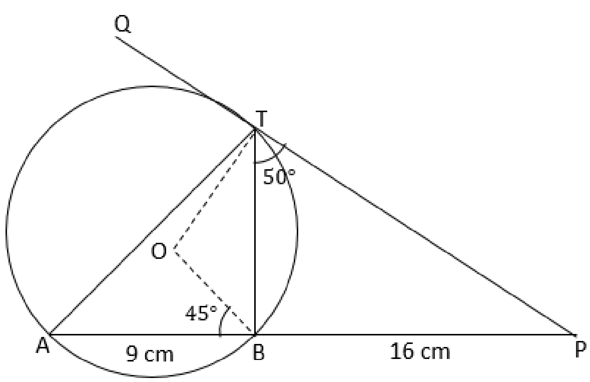
$-4y - 4 = 5x - 5$

$-4y - 5x = -1$

$5x + 4y = 1$

Therefore, the equation of line AD is $5x + 4y = 1$

(iii) In the given figure, O is the centre of the circle. PQ is a tangent to the circle at T. Chord AB produced meets the tangent at P.



$AB = 9\ cm, BP = 16\ cm, \angle PTB = 50^{\circ}$

$\angle OBA = 45^{\circ}$

Find:

(a) length of PT

(b) $\angle BAT$

(c) $\angle BOT$

(d) $\angle ABT$

[4]

Answer: $PT = 15cm$, $\angle BAT = 50°$, $\angle BOT = 100^{\circ}$, $\angle ABT = 85^{\circ}$

Explanation:

(a) Length of PT
We know that

$AB \times AP = PT^2$

$9 \times 25 = PT^2$

$PT = 15cm$

(b) Radius makes 90° at Tangent we have $\angle BTP = 50^\circ$

So, $\angle BTO = 90^\circ - 50^\circ \Rightarrow 40^\circ$

$\because \triangle BOT$ is isosceles $\triangle$

$\therefore \angle TBO = 40^\circ$

AID, $\triangle AOB$ is isosceles $\triangle$

$\therefore \angle OAB = 45^\circ \therefore \angle AOB = 90^\circ$

then $\angle AOT = 360^\circ - \left(90^\circ + 100^\circ\right) = 170^\circ$

$OA = OT$ (Radius)

$\therefore \triangle AOT$ is isosceles $\triangle$

$\therefore \angle OAT = \angle OTA = 5^\circ$

Now, $\angle BAT = \angle DAB + \angle OAT$

$= 45^\circ + 5^\circ$

$\angle BAT = 50°$

(c) $\angle BOT = 180^\circ - (\angle TBO + OTB)$

$= 180^\circ - 80^\circ$

$\angle BOT = 100^\circ$

(d) $\angle ABT = \angle ABO + \angle OBT$

$= 45^{\circ} + 40^{\circ}$

$\angle ABT = 85^{\circ}$

Question 5

(i) Mrs. Arora bought the following articles from a departmental store:

S.No.	Item	Price	Rate of GST	Discount
1.	Hair oil	Rs. 1200	18%	Rs. 100
2.	Cashew nuts	Rs. 600	12%	-

Find the:

(a) Total GST paid.

(b) Total bill amount including GST.

[3]

Explanation:

S.No.	Item	Price	Rate of GST	Discount	Discounted value
1.	Hair oil	Rs. 1200	18%	Rs. 100	Rs. 1100
2.	Cashew nuts	Rs. 600	12%	-	Rs. 600

Hair Oil:

CGST = 9% of 1100 = Rs. 99

SGST = 9% of 1100 = Rs. 99

Total GST = Rs. 198

Total amount = 1100 + 198 = Rs. 1298

Cashew Nuts:

CGST = 6% of 600 = Rs. 36

SGST = 6% of 600 = Rs. 36

Total GST = Rs. 72
Total amount = 600 + 72 = Rs. 672

Total GST = GST of Hair oil + GST of Cashew Nuts
= Rs. 198 + Rs. 72
= Rs. 270
Total amount = Rs 1298 + Rs. 672
= Rs. 1970

(ii) Solve the following inequation. Write down the solution set and represent it on the real number line.

$-5(x-9) \geq 17 - 9x > x + 2, x \in R$

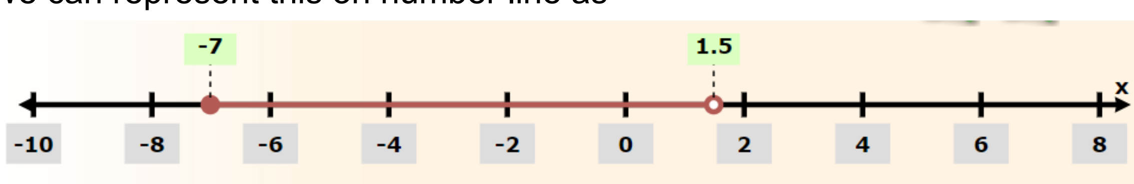
[3]

Answer: $x \in$ [-7, 1.5)
Explanation:
To solve this inequality, we'll first simplify it:

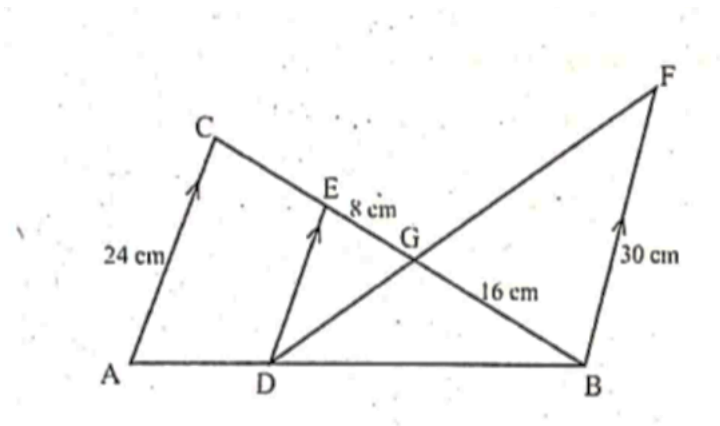
$$-5(x-9) \geq 17 - 9x > x + 2$$

-5 x + 45 ≥ 17 - 9 x > x + 2 (distribute -5 on the left and simplify)
-5 x + 45 ≥ 17 - 9 x and 17 - 9 x > x + 2 (split into two inequalities)
4 x ≥ - 28 and 15 > 10 x (add 9 x and subtract 17 from both sides of each inequality)
x ≥ - 7 and x < 1.5 (divide both sides of each inequality by 4 and 10, respectively)

To find the solution set for the given inequality x ≥ -7 and x < 1.5, we need to find the values of x that satisfy both conditions simultaneously.
x ≥ -7 means that x can take any value greater than or equal to -7.
x < 1.5 means that x can take any value less than 1.5.
To satisfy both conditions simultaneously, x must be a value that is greater than or equal to -7 AND less than 1.5.
Therefore, the solution set for x is:
$x \in$ [-7, 1.5)
We can represent this on number line as



(iii) In the given figure, $AC \| DE \| BF$.
If $AC = 24\ cm, EG = 8\ cm,\ GB = 16\ cm, BF = 30\ cm.$ [4]



(a) Prove $\triangle GED \sim \triangle GBF$
(b) Find DE
(c) $DB: AB$

[4]

Explanation:

Given: $AC \| DE \| BF$

$AC = 24\ cm,\ EG = 8\ cm,\ GB = 16\ cm,\ BF = 30\ cm$

(i) Prove: $\Delta GED \sim \Delta GBF$

Explanation: From fig, we have

$\angle EGD = \angle BGF$ (Vertically Opposite Angles)

$\angle DEG = \angle FBG$ (Alternate Interior Angles)

$\therefore \Delta GED \sim \Delta GBF$ (AA similarity criterion)

(ii) Find DE

Explanation:

$\because \Delta GED \sim \Delta GBF$

$\Rightarrow \frac{GE}{GB} = \frac{DE}{FB}$

$\Rightarrow \frac{8}{16} = \frac{DE}{30}$

$\Rightarrow DE = \frac{1}{2} \times 30 = 15\ cm$

(iii) In figure, we have $ED \parallel AC$

∴ We have,

$\Delta BED \sim \Delta BCA$ (AA similarity)

$\Rightarrow \frac{DB}{DE} = \frac{AB}{AC}$

$\Rightarrow \frac{DB}{AB} = \frac{15}{24}$

$\Rightarrow DB : AB = 5 : 8$

Question 6

(i) The following distribution gives the daily wages of 60 workers of a factory.

Daily income in ₹	Number of workers (f)
$200 - 300$	6
$300 - 400$	10
$400 - 500$	14
$500 - 600$	16
$600 - 700$	10
$700 - 800$	4

Use graph paper to answer this question.
Take $2\ cm = ₹100$ along one axis and $2\ cm = 2$ workers along the other axis.
Draw a histogram and hence find the mode of the given distribution.

[3]

Explanation:

To draw a histogram for the given distribution, we will use the following steps:

Step 1: Draw the x and y-axis with suitable scales on the graph paper. We will take 2 cm = ₹100 along the x-axis and 2 cm = 2 workers along the y-axis.

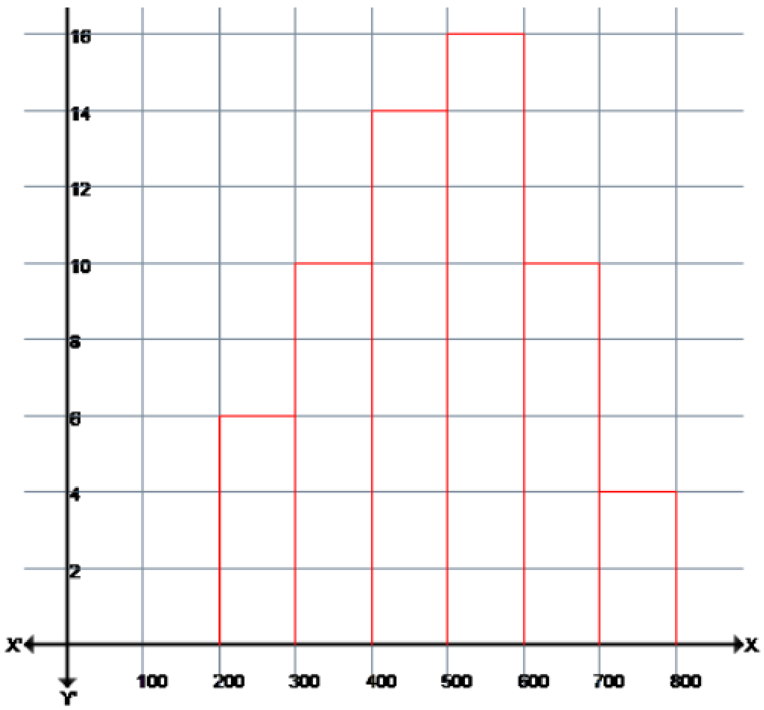
Step 2: Mark the class intervals on the x-axis and the corresponding frequencies on the y-axis.

Step 3: Draw rectangles for each class interval with the base of the rectangle on the x-axis and the height equal to the corresponding frequency on the y-axis.

Step 4: The histogram is obtained by placing the rectangles adjacent to each other.

Class	Lower	Upper	Frequency f
200 - 300	200	300	6
300 - 400	300	400	10
400 - 500	400	500	14
500 - 600	500	600	16
600 - 700	600	700	10
700 - 800	700	800	4

Using the above steps, the histogram for the given distribution is as follows:



Histogram for the given distribution

To find the mode of the given distribution, we look for the class interval with the highest frequency. From the histogram, we can see that the class interval 500-600 has the highest frequency of 16. Therefore, the mode of the given distribution is ₹500-₹600.

(ii) The 5^{th} term and the 9^{th} term of an Arithmetic Progression are 4 and -12 respectively. Find:
(a) the first term
(b) common difference
(c) sum of 16 terms of the AP.

[3]

Explanation:

Given 5^{th} term $= 4$

$9^{th}\ term\ = -\ 12$

Let a is the first term and d is the common difference of the A.P.

$t_5 \Rightarrow a + 4d = 4 - \ (i)$

$t_9 \Rightarrow a + 8d =- 12 - \ (ii)$

By subtracting (ii) - (i) we get -

$4d = \ - 16$

$d = \ - 4$

Now substitute the value of d in (1) eq we get -

$a + 4(- 4) = 4$

$- 16 + \ a = 4$

$a = 4 + 16 = 20$

Common difference is -4

Sum of 16 terms can be given by,

$S_{16} = = \frac{16}{2}[2\times20 + 15(- 4)]$

$S_{16} \ = 8[40 - 60]$

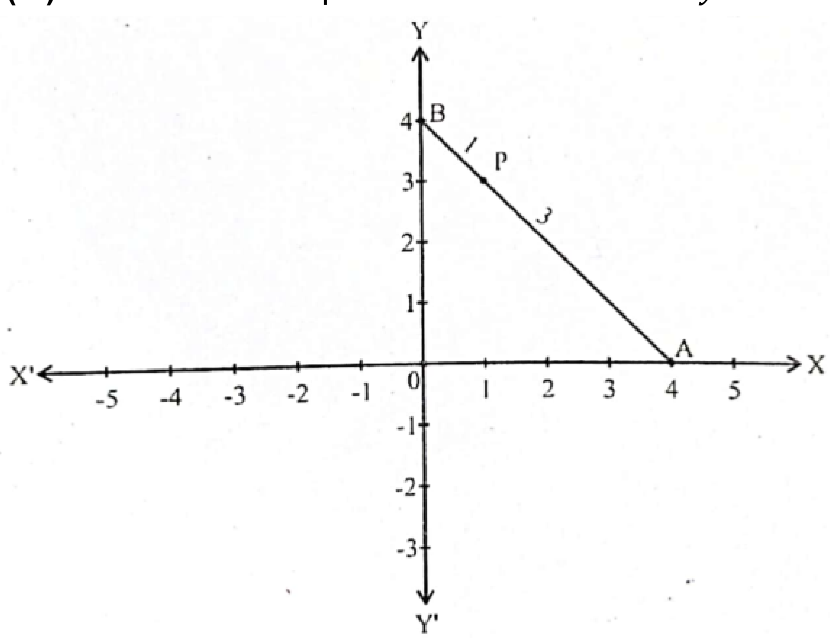
$S_{16} \ = \ - 160$

(a) First term = 20

(b) Common difference = - 4

(c) Sum of 16 terms = - 160

(iii) A and B are two points on the x-axis and y-axis respectively.



(a) Write down the coordinates of A and B.
(b) P is a point on AB such that $AP: PB = 3: 1$. Using section formula find the coordinates of point P.
(c) Find the equation of a line passing through P and perpendicular to AB.

[4]

Explanation:
(a) Co-ordinates of $A = (4, 0)$
co-ordinates of $B = (0, 4)$
(b) The coordinates of the point $P(x, y)$ which divides the line segment joining points $A(x_1, y_1)$ and $B(x_2, y_2)$ internally in the ratio $m: n$ are

$$\left(\frac{mx_2+nx_1}{m+n}, \frac{my_2+ny_1}{m+n}\right)$$

Here -

$$m = 3 \ n = 1 \ x_1 = 4 \ x_2 = 0 \ y_1 = 0 \ y_2 = 4$$

$$x = \frac{mx_2+nx_1}{m+n}, y = \frac{my_2+ny_1}{m+n}$$

$$x = \frac{3\times0+1\times4}{3+1}, \ y = \frac{3\times4+1\times0}{3+1}$$

$$x = \frac{4}{4}, \ y = \frac{12}{4}$$

x = 1, y = 3
Co-ordinates of P is $(1, 3)$

(c) Equation of line connecting AB formula:- $\left(y - y_1\right) = \frac{\left(y_2-y_1\right)}{\left(x_2-x_1\right)}\left(x - x_1\right)$

$$x_1 = 4 \, x_2 = 0 \ y_1 = 0 \, y_2 = 4$$

Equation of line segment will be -

$$\Rightarrow (y - 0) = \frac{(4-0)}{(0-4)}(x - 4)$$

$$\Rightarrow y = \frac{4}{-4}(x - 4) \quad \Rightarrow \quad -y = x - 4 \Rightarrow y = 4 - x$$

Comparing with $y = mx + 6$ we get the gradient is -1.
So the gradient of the line perpendicular to this line will be 1.
Now equation of the line parsing throng $P(1, 3)$ and $\perp$ to AB will be -

$$\Rightarrow y - y_1 = m\left(x - x_1\right) \ \Rightarrow (y - 3) = 1(x - 1)$$

$$\Rightarrow y - 3 = x - 1 \quad \Rightarrow y = x - 1 + 3 \quad \Rightarrow y = x + 2$$

Question 7

(i) A bag contains 25 cards, numbered through 1 to 25 . A card is drawn at random. What is the probability that the number on the card drawn is:

(a) multiple of 5

(b) a perfect square

(c) a prime number?

[3]

Explanation:

(a) Cards which are multiple of 5 will be - 5, 10, 15, 20, 25

Probability of drawn card to be multiple of 5 will be given by -

$$\Rightarrow P = \frac{5}{25} = \frac{1}{5}$$

$$P = \frac{No\ of\ cards\ multiple\ of\ 5}{total\ No\ of\ cards}$$

$$P = \frac{5}{25} \quad P = \frac{1}{5}$$

(b) Perfect numbers in 1 to 25 are $1, 4, 9, 16, 25$

Probability will be given by

$$P = \frac{No\ of\ perfect\ Squares}{total\ Numbers}$$

$$p = \frac{5}{25} = \frac{1}{5}$$

(c) Prime Numbers in 1 to 25 are $2, 3, 5, 7, 11, 13, 17, 19, 23$

Probability will be given by -

$$P = \frac{Number\ of\ Primes}{Total\ Numbers} \quad P = \frac{9}{25}$$

(ii) A man covers a distance of 100 km, travelling with a uniform speed of x km/hr. Had the speed been 5 km/hr more it would have taken 1 hour less. Find x the original speed.

[3]

Explanation:

Let the usual speed of train be km/hr.

The increased speed of the train $= (x + 5) km/hr$

Time taken by the train under usual speed to cover $100\ km = \frac{100}{x} hr$

Time taken by the train under increased speed to cover $100\ km = \frac{100}{x+5} hr$

Therefore,

$$\frac{100}{x} - \frac{100}{(x+5)} = \frac{60}{60}$$

$$\frac{100(x+5)-100x}{x(x+5)} = 1$$

$$\frac{100x+500-100x}{x^2+5x} = 1$$

$$500 = x^2 + 5x$$

$$x^2 + 5x - 500 = 0$$

$$x^2 + 25x - 20x - 500 = 0$$

$$x(x + 25) - 20(x + 25) = 0$$

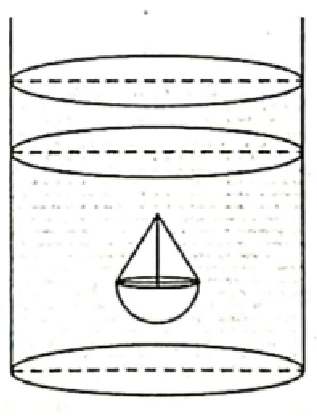
$$(x + 25)(x - 20) = 0$$

$$x = - 25 \text{ or } x = 20$$

Since, the speed of the train can never be negative

Therefore, the original speed of the train is $20\ km/hr$.

(iii) A solid is in the shape of a hemisphere of radius $7\ cm$, surmounted by a cone of height $4\ cm$. The solid is immersed completely in a cylindrical container filled with water to a certain height. If the radius of the cylinder is $14\ cm$, find the rise in the water level.



[4]

Answer: 1.5 cm

Explanation:

First, we need to find the volume of the hemisphere and the cone, then we can add them together to find the total volume of the solid. Let's begin by finding the volume of the hemisphere.
The volume of a hemisphere of radius r is given by:

$$V = \frac{2}{3}\pi r^3$$

Substituting r = 7, we get:

$$V_1 = \frac{2}{3}\pi r^3 = \frac{2}{3} \times \frac{22}{7} \times 7 \times 7 \times 7 = 718.66\ cm^3$$

Next, let's find the volume of the cone. The formula for the volume of a cone is

$$V = \frac{1}{3}\pi r^2 h$$

where r is the radius of the base and h is the height. Substituting r = 7 and h = 4, we get

$$V_2 = \frac{1}{3} \times \frac{22}{7} \times 7 \times 7 \times 4 = 205.33\ cm^3$$

The total volume of the solid is:

V = V_1 + V_2 = 718.66 + 205.33 = 923.99 cm^3

Now, let's find the rise in the water level. We know that the volume of water displaced is equal to the volume of the solid. Let h be the height by which the water level rises when the solid is immersed in the water. The formula for the volume of a cylinder is:

$$V = \pi r^2 h$$

Substituting r = 14 and V = 923.99, we get

$$923.99 = \frac{22}{7} \times 14 \times 14 \times h$$

Solving for h, we get:
h = 1.49 ≈ 1.5 cm
Therefore, the water level rises by approximately 1.5 cm when the solid is completely immersed in the cylindrical container.

Question 8

(i) The following table gives the marks scored by a set of students in an examination. Calculate the mean of the distribution by using the short cut method.

Marks	Number of Students (f)
0 – 10	3
10 – 20	8
20 – 30	14
30 – 40	9
40 – 50	4
50 – 60	2

[3]

Answer: 27.25

Explanation:

Class (1)	Frequency (f) (2)	Mid value (x) (3)	$d = \frac{x - A}{h} = \frac{x - 35}{10}$ $A = 35, h = 10$ (4)	$f \cdot d$ (5) = (2) × (4)
0 - 10	3	5	-3	-9
10 - 20	8	15	-2	-16
20 - 30	14	25	-1	-14
30 - 40	9	35=A	0	0
40 - 50	4	45	1	4
50 - 60	2	55	2	4
---	---	---	---	---
	$n = 40$	-----	-----	$\sum f \cdot d = -31$

$$Mean\ x^{-} = A + \frac{\sum fd}{n} \cdot h$$

$$= 35 + \frac{-31}{40} \cdot 10$$

$$= 35 + (-\ 0.775) \cdot 10$$

$$= 35 - 7.75$$

$$= 27.25$$

(ii) What number must be added to each of the numbers $4, 6, 8, 11$ in order to get the four numbers in proportion?

[3]

Explanation:
Let we need to add x to make these numbers in proportion then -
According to the question -

$$(4 + x):(6 + x)::(8 + x):(1 + x)$$

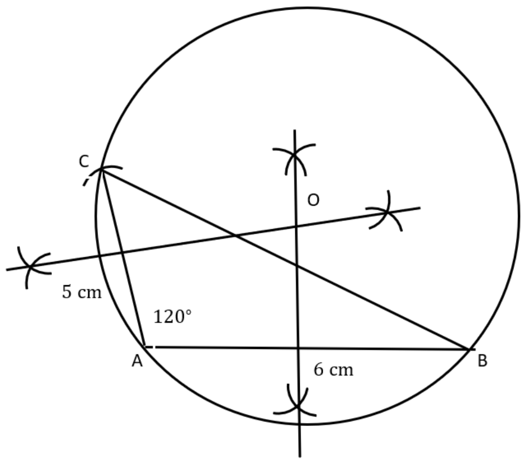
$$\Rightarrow (4 + x)(11 + x) = (8 + x)(6 + x) \Rightarrow 44 + 4x + 11x + x^2 = 48 + 8x + 6x + x^2 \Rightarrow 44$$

So we need to add 4 in all the numbers to get them in proportion.

(iii) Using ruler and compass construct a triangle ABC in which $AB = 6\ cm, \angle BAC = 120^{\circ}$ and $= 5\ cm$. Construct a circle passing through A, B and C. Measure and write down the radius of the circle.

[4]

Answer: 5.5 cm
Explanation:
Steps of construction:
1. Draw a line segment AB of length 6 cm using a ruler.
2. At point A, construct an angle of 120 degrees using a compass.
3. From point A, draw a line segment AC of length 5 cm that intersects the angle at point C.
4. Connect points B and C with a line segment to form triangle ABC.
5. To construct the circumcircle of the triangle, place the compass on any point on the triangle and draw a circle that passes through all three vertices (A, B, and C). This circle is the circumcircle of the triangle.



By measuring the length of the radius it is 5.5 cm.

Question 9

(i) Using Componendo and Dividendo solve for x;

$$\frac{\sqrt{2x+2}+\sqrt{2x-1}}{\sqrt{2x+2}-\sqrt{2x-1}} = 3$$

[3]

Explanation:

We begin by simplifying the given equation using componendo and dividendo as follows:

$$\frac{\sqrt{2x+2}+\sqrt{2x-1}}{\sqrt{2x+2}-\sqrt{2x-1}} = 3$$

Adding 1 to both the numerator and denominator of the left-hand side, we get:

$$\frac{(\sqrt{2x+2}+\sqrt{2x-1})+(\sqrt{2x+2}-\sqrt{2x-1})}{\sqrt{2x+2}-\sqrt{2x-1}+1} = \frac{4}{2}$$

Simplifying the expression further, we get:

$$\frac{2\sqrt{2x+2}}{\sqrt{2x+2}-\sqrt{2x-1}+1} = 2$$

Multiplying both sides by $\sqrt{2x+2} - \sqrt{2x-1} + 1$, we get:

$2\sqrt{2x+2} = 2(\sqrt{2x+2} - \sqrt{2x-1} + 1)$

Simplifying, we get:

$\sqrt{2x+2} = (\sqrt{2x+2} - \sqrt{2x-1} + 1)$

Subtracting $\sqrt{2x+2}$ from both sides, we get:

$-\sqrt{2x-1} + 1 = 0$

Adding $\sqrt{2x-1}$ to both sides, we get:

$1 = \sqrt{2x-1}$

Squaring both sides, we get:

$1^2 = (\sqrt{2x-1})^2$

$1 = 2x - 1$

$2 = 2x$

$x = 1$

Therefore, the solution to the equation $\frac{\sqrt{2x+2}+\sqrt{2x-1}}{\sqrt{2x+2}-\sqrt{2x-1}} = 3$ is $x = 1$.

(ii) Which term of the Arithmetic Progression (A.P.) 15, 30, 45, 60... is 300 ? Hence find the sum of all the terms of the Arithmetic Progression (A.P.)

[3]

Answer: 20th term, 3150

Explanation:

To find the term of an Arithmetic Progression (A.P.), we use the formula:

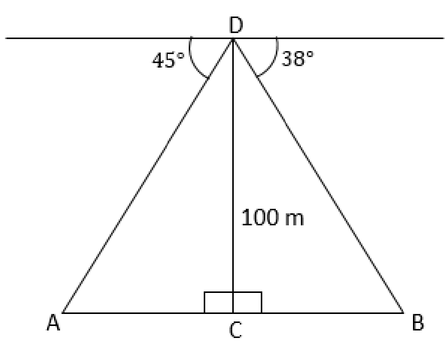
$a_n = a_1 + (n-1)d$

where a_n is the nth term of the A.P., a_1 is the first term of the A.P., n is the number of terms, and d is the common difference between consecutive terms.

Here, we have the first term a_1 = 15, and the common difference d = 30 - 15 = 15.

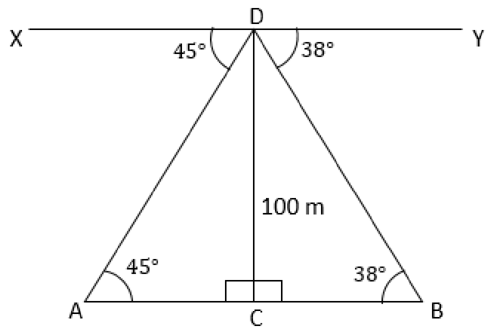
We need to find the term of the A.P. which is equal to 300. So, we substitute these values into the formula:
300 = 15 + (n-1)15
300 - 15 = 15(n-1)
285 = 15(n-1)
n-1 = 19
n = 20
Therefore, the 20th term of the A.P. is 300.
To find the sum of all the terms of an A.P., we use the formula:
S_n = (n/2)(a_1 + a_n)
where S_n is the sum of the first n terms of the A.P.
Substituting the values, we get:
S_n= (20/2)(15 + 300)
S_n = 10(315)
S_n = 315
Therefore, the sum of all the terms of the A.P. is 3150.

(iii) From the top of a tower $100\ m$ high a man observes the angles of depression of two ships A and B, on opposite sides of the tower as 45° and 38° respectively. If the foot of the tower and the ships are in the same horizontal line find the distance between the two ships A and B to the nearest metre.
(Use Mathematical Tables for this question.)



[4]

Answer: 228.04 m
Explanation:



In the figure,
A and B are the positions of the ships A and B.
CD = 100 m.
The angles of depression are ∠DAC and ∠DBC.
Since the line XY is parallel to AB, we have
∠YDB = ∠DBC = 38° and ∠XDA = ∠DAC = 45°
To find: Distance between the two ships
From right △ACD, tan 45° = CD/AC
1 = 100/AC
or AC = 100
From right △BCD:
tan 38° = CD/CB
0.781 = 100/BC
or BC = 128.04
Now, AB = AC + CB
= 100 + 128.04
= 228.04
Hence distance between the ships is 228.04 meters.

Question 10

(i) Factorize completely using factor theorem: $2x^3 - x^2 - 13x - 6$

[4]

Explanation:
For factorizing this cubic expression, no identity is useful.
Thus, we need to use factorization by regrouping terms.
We can write, $-x^2 = x^2 - 2x^2$ and $-13x = -x - 12x$

$$\Rightarrow 2x^3 - x^2 - 13x - 6 = 2x^3 + x^2 - 2x^2 - x - 12x - 6$$

We have, $-2x^2 = (-x)\times 2x$ and $-12x = (-6)\times 2x$

$$\Rightarrow 2x^3 - x^2 - 13x - 6 = 2x^3 + x^2 + (-x)\times 2x - x + (-6)\times 2x - 6$$

Observe that x^2 is common for the first two terms, $-x$ is common for the next two terms and -6 is common for the last two terms.

$$\Rightarrow 2x^3 - x^2 - 13x - 6 = x^2(2x + 1) + (-x)(2x + 1) + (-6)(2x + 1)$$

Now, $(2x + 1)$ is the common term.

$$\therefore 2x^3 - x^2 - 13x - 6 = (2x + 1)\left(x^2 - x - 6\right)$$

So, one factor of the given expression is $(2x + 1)$. Now, we need to factorize $\left(x^2 - x - 6\right)$.

For factorizing, this expression, split the middle term in such a way that the product of the coefficients of the new terms is equal to the product of the coefficients of the first and last terms in the expression.

Here, product of co-effs of first and last terms = $1\times(-6) = -6$

So, if the middle term $-x$ is split into two terms say ax, bx, then $a + b = -1$ and $ab = -6$.

Observe that values -3 and 2 satisfy these equations.

$$\Rightarrow x^2 - x - 6 = x^2 - 3x + 2x - 6$$

Observe that x is common for the first two terms and 2 is common for the next two terms.

$$\Rightarrow x^2 - 3x + 2x - 6 = x(x - 3) + 2(x - 3)$$

Now, $(x - 3)$ is the common term.

$$\Rightarrow x^2 - x - 6 = (x - 3)(x + 2)$$

Thus, the factors of $x^2 - x - 6$ are $(x - 3)$ and $(x + 2)$.

$$\therefore 2x^3 - x^2 - 13x - 6 = (2x + 1)(x - 3)(x + 2)$$

Hence, the factors of $2x^3 - x^2 - 13x - 6$ are $(2x + 1), (x - 3)$ and $(x + 2)$.

(ii) Use **graph paper** to answer this question.

During a medical checkup of 60 students in a school, weights were recorded as follows:

Weight (in kg)	Number of Students
$28 - 30$	2
$30 - 32$	4
$32 - 34$	10
$34 - 36$	13
$36 - 38$	15
$38 - 40$	9
$40 - 42$	5
$42 - 44$	2

Taking $2\ cm = 2\ kg$ along one axis and $2\ cm = 10$ students along the other axis draw an ogive. Use your graph to find the:

(a) median

(b) upper Quartile

(c) number of students whose weight is above $37\ kg$.

[6]

Explanation:

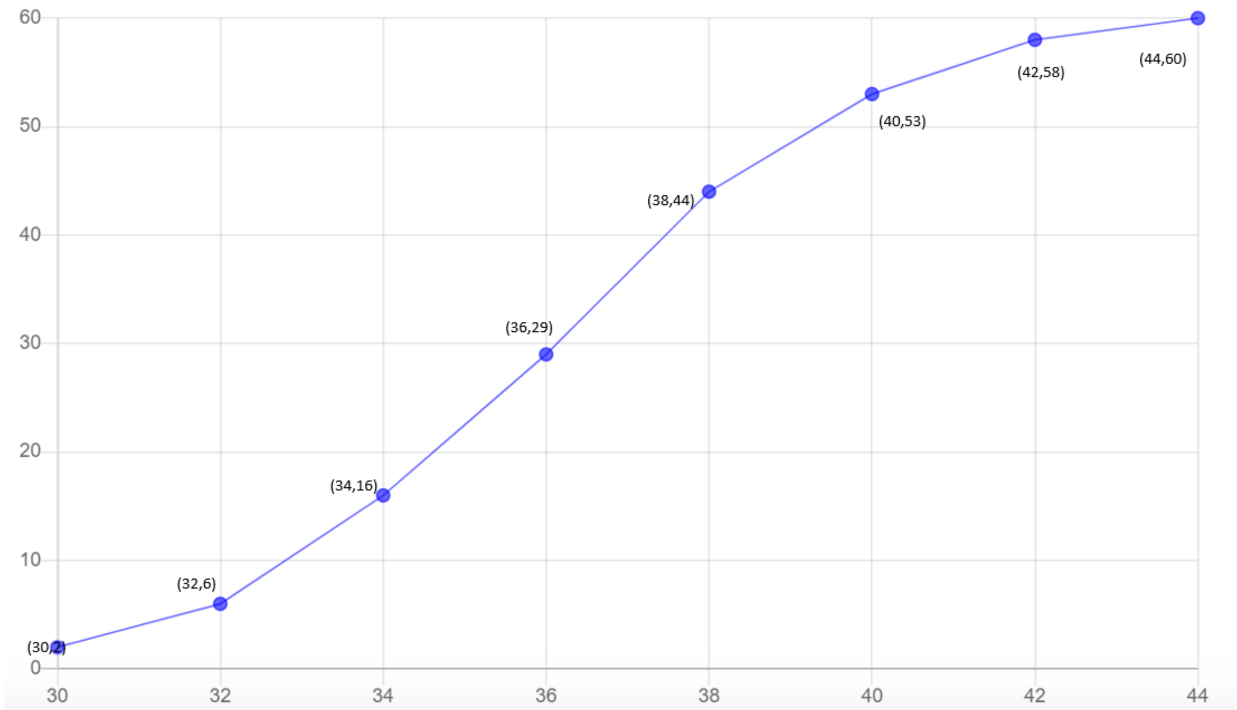
To draw the ogive, we first need to calculate the cumulative frequencies. The cumulative frequency is the total number of students up to and including a certain weight.

Weight (in kg)	Number of Students (f)	Cumulative Frequency (cf)
28 – 30	2	2
30 – 32	4	6
32 – 34	10	16
34 – 36	13	29
36 – 38	15	44
38 – 40	9	53
40 - 42	5	58
42 - 44	2	60

We can now plot the cumulative frequencies on the graph paper. The horizontal axis represents the weight (in kg), and the vertical axis represents the cumulative frequency (in number of students).

We use 2cm=2kg along the horizontal axis and 2cm=10 students along the vertical axis.

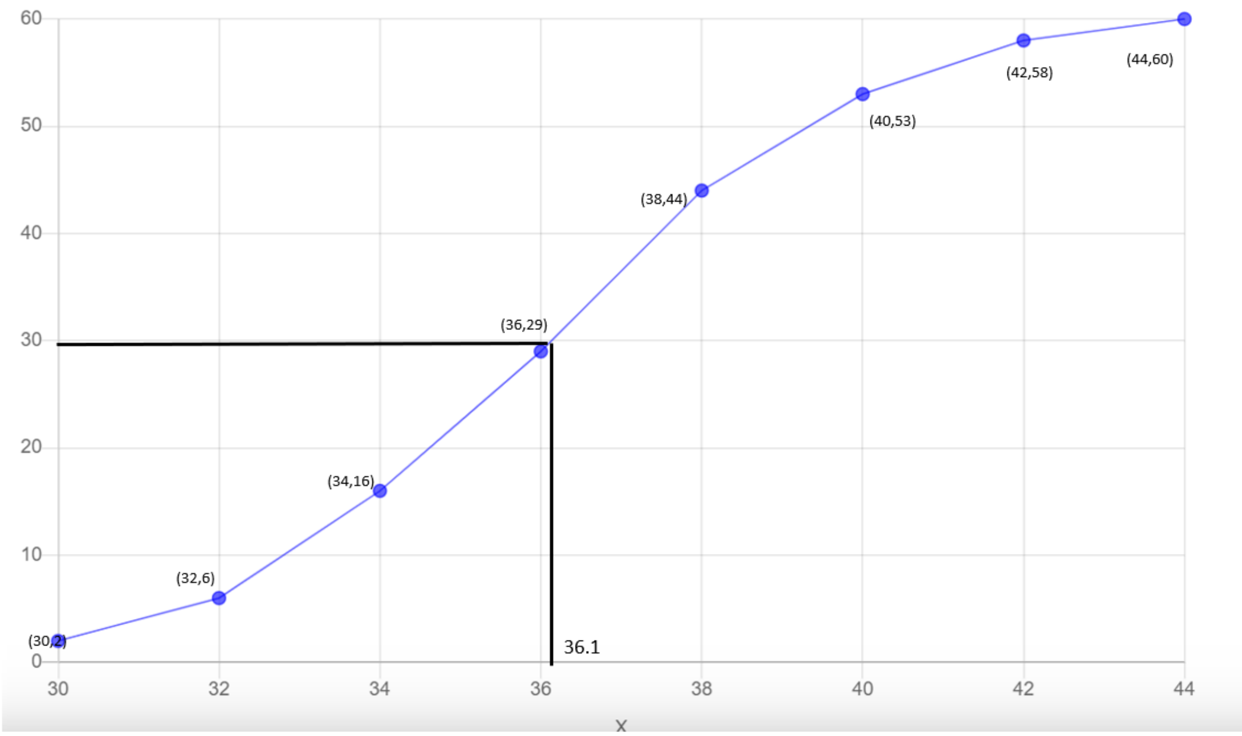
Ogive graph for weight distribution



(a) Using the graph drawn, we get

Median = $\left(\frac{n}{2}\right)^{th}$ term, where $n = 60$

= 30^{th} term = 36.15 kg (Approximately)



(b) Upper quartile (Q_3) :

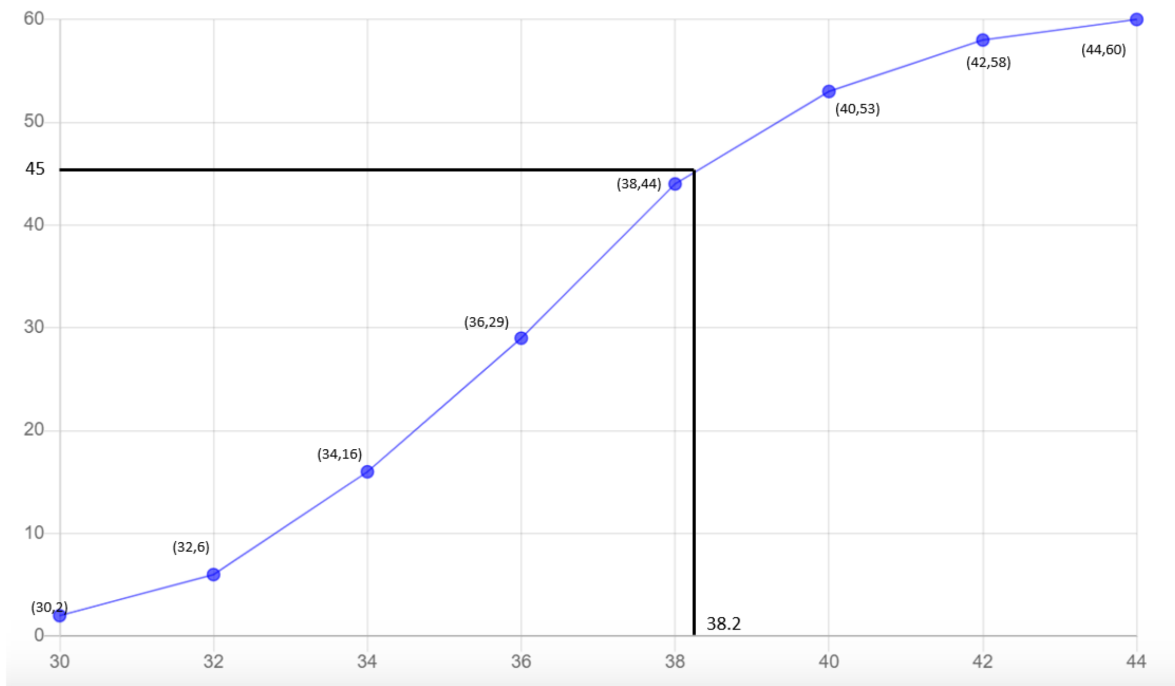
Here, $n = 60$

Q_3 class : $\left(\frac{3n}{4}\right)^{th}$

$= \left(\frac{3 \cdot 60}{4}\right)^{th}$ term

$= (45)^{th}$ term

= 38.2



(c) The number of students whose weight is above 37 kg is the total number of students (60) minus the cumulative frequency at the weight of 37 kg (which is 29). Therefore, the number of students whose weight is above 37 kg is:

60 - 29 = 31 students.

Mathematics [Official]

CISCE

Academic Year: 2023-2024

(English Medium)

Date & Time: 15th March 2024, 11:00 am

Duration: 2h30m **Marks: 80**

1. Answers to this Paper must be written on the paper provided separately.
2. You will not be allowed to write during the first 15 minutes.
3. This time is to be spent reading the question paper.
4. The time given at the head of this Paper is the time allowed for writing the answers.
5. Attempt all questions from Section A and any four questions from Section B.
6. All work, including rough work, must be clearly shown and must be done on the same sheet as the rest of the Solution.
7. Omission of essential work will result in a loss of marks.
8. The intended marks for questions or parts of questions are given in brackets [ ].
9. Mathematical tables and graph papers are provided.

SECTION-A (40 Marks) (Attempt all questions from this Section)

Question 1. Choose the correct Solutions to the questions from the given options. (Do not copy the questions. Write the correct Solutions only.)

1.1. For an Intra-state sale, the CGST paid by a dealer to the Central government is ₹ 120. If the marked price of the article is ₹ 2000, the rate of GST is ______.

1. 6%
2. 10%
3. **12%**
4. 16.67%

Solution

For an Intra-state sale, the CGST paid by a dealer to the Central government is ₹ 120. If the marked price of the article is ₹ 2000, the rate of GST is 12%.

Explanation:

CGST paid = ₹ 120

M.P. of article ₹ 2,000

In case of intra-state sales,

CGST = SGST

And GST amount = CGST + SGST

= 120 + 120

= 240

Then, GST Rate = $\frac{\text{GST Amount}}{\text{Marked Price}} \times 100$

$= \frac{240}{2000} \times 100$

= 12%

1.2. What must be subtracted from the polynomial $x^3 + x^2 - 2x + 1$, so that the result is exactly divisible by $(x - 3)$?

1. 31
2. – 30
3. 30
4. 31

Solution

31

Explanation:

On dividing $x^3 + x^2 - 2x + 1$ by $(x - 3)$, we get
Put x = 3, then by remainder theorem

$P(3) = 3^3 + 3^2 - 2 \times 3 + 1$

$= 27 + 9 - 6 + 1$

$= 36 - 6 + 1$

$= 31$

So, 31 must be subtracted in order to divide p(x) by (x – 3).

1.3. The roots of the quadratic equation px2 – qx + r = 0 are real and equal if ______.

1. p2 = 4qr
2. q2 = 4pr
3. – q2 = 4pr
4. p2 > 4pr

Solution

The roots of the quadratic equation $px^2 - qx + r = 0$ are real and equal if **$q^2 = 4pr$.**

Explanation:

Given, equation is $px^2 - qx + r = 0$

On comparing it with $ax^2 + bx + c = 0$, we get

$a = p, b = -q, c = r$

For roots to be equal, D = 0

i.e., $b^2 - 4ac = 0$

$\Rightarrow (-q)^2 - 4 \times p \times r = 0$

$\Rightarrow q^2 - 4pr = 0$

$\Rightarrow q^2 = 4pr$

1.4. If matrix A = $\begin{bmatrix} 2 & 2 \\ 0 & 2 \end{bmatrix}$ and A2 = $\begin{bmatrix} 4 & x \\ 0 & 4 \end{bmatrix}$ then the value of x is ______.

1. 2
2. 4
3. 8
4. 10

Solution

If matrix A = $\begin{bmatrix} 2 & 2 \\ 0 & 2 \end{bmatrix}$ and A^2 = $\begin{bmatrix} 4 & x \\ 0 & 4 \end{bmatrix}$, then the value of x is **8**.

Explanation:

Given, matrix A = $\begin{bmatrix} 2 & 2 \\ 0 & 2 \end{bmatrix}$.

Then A^2 = $\begin{bmatrix} 2 & 2 \\ 0 & 2 \end{bmatrix}\begin{bmatrix} 2 & 2 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 4 & 8 \\ 0 & 4 \end{bmatrix}$

On comparing it with A^2 = $\begin{bmatrix} 4 & x \\ 0 & 4 \end{bmatrix}$, we get that

x = 8

1.5. The median of the following observations arranged in ascending order is 64. Find the value of x:

27, 31, 46, 52, x, x + 4, 71, 79, 85, 90

1. 60
2. 61
3. 62
4. 66

Solution

62

Explanation:

In the given data number of terms are = 10 ...(Even)

Then, median = $\dfrac{\frac{n}{2}\text{th term} + \left(\frac{n}{2}+1\right)^{\text{th}}\text{ term}}{2}$

But given medium = $\dfrac{5^{\text{th}}\text{ term} + 6^{\text{th}}\text{ term}}{2}$

= $\dfrac{x + x + 4}{2}$

= $\dfrac{2x + 4}{2}$

= x + 2

= x + 2

But given median = 64

On comparing,

∴ x + 2 = 64

⇒ x = 62

1.6. Points A(x, y), B(3, –2) and C(4, –5) are collinear. The value of y in terms of x is ______.

1. 3x – 11
2. 11 – 3x
3. 3x – 7
4. **7 – 3x**

Solution

Points A(x, y), B(3, –2) and C(4, –5) are collinear. The value of y in terms of x is 7 – 3x.

Explanation:

Points A(x, y), B(3, –2) and C(4, –5) are collinear.

Then, the area of the triangle formed by those points will be zero

$$\therefore \frac{1}{2}\begin{vmatrix} x & y & 1 \\ 3 & -2 & 1 \\ 4 & -5 & 1 \end{vmatrix} = 1$$

⇒ x(– 2 + 5) – y(3 – 4) + 1(– 15 + 8) = 0

⇒ 3x + y – 7 =0

⇒ y = 7 – 3x

1.7. The given table shows the distance covered and the time taken by a train moving at a uniform speed along a straight track:

Distance (in m)	60	90	y
Time (in sec)	2	x	5

The values of x and y are:

1. x = 4, y = 150
2. x = 3, y = 100
3. x = 4, y = 100
4. x = 3, y = 150

Solution

x = 3, y = 150

Explanation:

It is a directional change.

If the speed is uniform, the moving distance covered will be larger than the time taken then,

$$\Rightarrow \frac{60}{2} = \frac{90}{x} = \frac{y}{5}$$

$$\Rightarrow x = \frac{90 \times 2}{60} \text{ and } y = \frac{60 \times 5}{2}$$

$$x = \frac{180}{60} \text{ and } y = \frac{300}{2}$$

$$\therefore x = 3 \text{ and } y = 150$$

1.8. The 7th term of the given Arithmetic Progression (A.P.):

$\frac{1}{a}, \left(\frac{1}{a}+1\right), \left(\frac{1}{a}+2\right)$... is:

$\left(\frac{1}{a}+6\right)$

$\left(\frac{1}{a}+7\right)$

$\left(\frac{1}{a}+8\right)$

$\left(\frac{1}{a}+7^7\right)$

Solution

$$\left(\frac{\mathbf{1}}{\mathbf{a}}+\mathbf{6}\right)$$

Explanation:

Given A.P. is $\frac{1}{a}, \left(\frac{1}{a}+1\right), \left(\frac{1}{a}+2\right)$

Here, first term, A = $\frac{1}{a}$

Common difference D = $\frac{1}{a}+1-\frac{1}{a}$ = 1

Then, 7th term of A.P. = A + (n – 1)D

= $\frac{1}{a}+(7-1)\times 1$

= $\frac{1}{a}+6$

1.9. The sum invested to purchase 15 shares of a company of nominal value ₹ 75 available at a discount of 20% is ______.

1. ₹ 60
2. ₹ 90
3. ₹ 1350
4. **₹ 900**

Solution

The sum invested to purchase 15 shares of a company of nominal value ₹ 75 available at a discount of 20% is ₹ 900.

Explanation:

Number of shares purchased = 15

Market value of each share = $75-\frac{20}{100}\times 75$

= 75 – 15
= ₹ 60

Total money invested to purchase 15 shares

= 15 × 60

= ₹ 900

1.10. The circumcentre of a triangle is the point which is ______.

1. at equal distance from the three sides of the triangle.
2. **at equal distance from the three vertices of the triangle.**
3. the point of intersection of the three medians.
4. the point of intersection of the three altitudes of the triangle.

Solution

The circumcentre of a triangle is the point which is at equal distance from the three vertices of the triangle.

Explanation:
We know that,
The circumcenter of a triangle is equidistant from all three of its vertices.
This means that the distance from the circumcenter to each vertex is equal.

1.11. Statement 1: $\sin^2\theta + \cos 2\theta = 1$

Statement 2: $\operatorname{cosec}^2\theta + \cot 2\theta = 1$

Which of the following is valid?

1. **Only 1**
2. Only 2
3. Both 1 and 2
4. Neither 1 nor 2

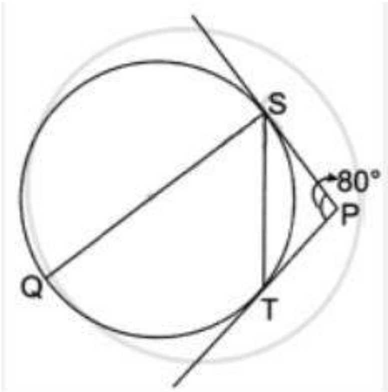
Solution

Only 1

Explanation:

From statement 2: $\mathrm{cosec}^2\theta - \cot^2\theta = 1$ is correct

1.12. In the given diagram, PS and PT are the tangents to the circle. SQ || PT and ∠SPT = 80°. The value of <QST is ______.



1. 140°
2. 90°
3. 80°
4. 50°

Solution

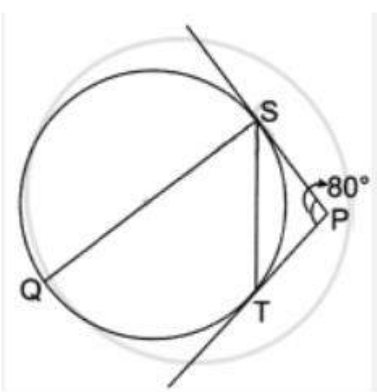
In the given diagram, PS and PT are the tangents to the circle. SQ || PT and <SPT = 80°. The value of <QST is 50°.

Explanation:

PS and PT are tangents from an exterior point to a circle from point P

i.e., PS = PT

So <PST = <PTS



In ΔPST,

<PST + <PTS + <SPT = 180°

2<PTS = 180° – 80° = 100°

<PTS = 50°

Here, SQ || PT and ST is a transversal

Then, <QST = <STP = 50° ...(Alternate pair of angles)

1.13. Assertion (A): A die is thrown once and the probability of getting an even number is $\frac{2}{3}$.

Reason (R): The sample space for even numbers on a die is {2, 4, 6}.

1. A is true, R is false.
2. **A is false, R is true.**
3. Both A and R are true.
4. Both A and R are false.

Solution

A is false, R is true.

Explanation:

In assertion, when a dice is thrown the total outcomes = 6
Even numbers = {2, 4, 6} i.e. 3

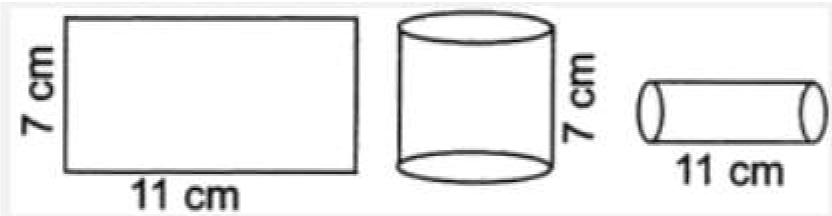
$$\text{Required probability} = \frac{3}{6} = \frac{1}{2}$$

So, assertion is false

In reason part, the even number on a dice is {2, 4, 6}

So, reason is true.

1.14. A rectangular sheet of paper of size 11 cm × 7 cm is first rotated about the side 11 cm and then about the side 7 cm to form a cylinder, as shown in the diagram. The ratio of their curved surface areas is ______.

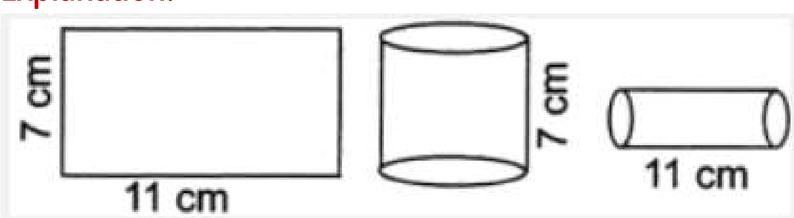


1. 1 : 1
2. 7 : 11
3. 11 : 7
4. $\frac{11\pi}{7} : \frac{7\pi}{11}$

Solution

A rectangular sheet of paper of size 11 cm × 7 cm is first rotated about the side 11 cm and then about the side 7 cm to form a cylinder, as shown in the diagram. The ratio of their curved surface areas is 1 : 1.

Explanation:



In first case, height of cylinder (h) = 7 cm

Circumference of cylinder (2πr) = 11

Then, curved surface area

C1 = 2πrh

= 11 × 7

= 77 cm

In second case, height of cylinder (H) = 11 cm

Circumference of cylinder ($2\pi R$) = 7

Then, curved surface area,

$C2 = 2\pi RH$

$= 7 \times 11$

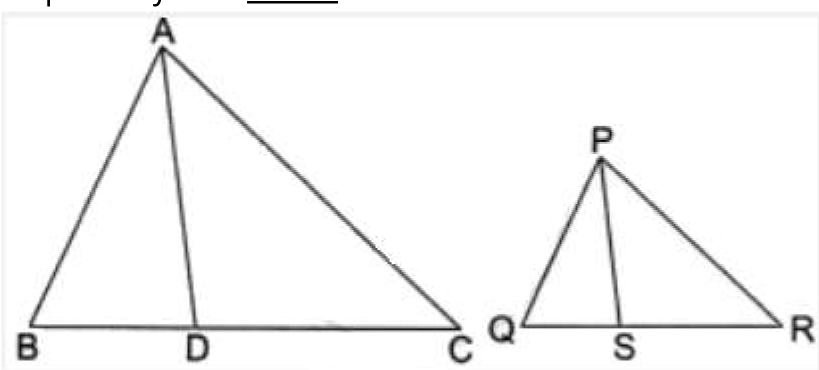
= 77 cm

Then, $C_1 : C_2$

= 77 : 77

= 1 : 1

1.15. In the given diagram, ΔABC ~ ΔPQR. If AD and PS are bisectors of <BAC and <QPR respectively then ______.



ΔABC ~ ΔPQS

ΔABD ~ ΔPQS

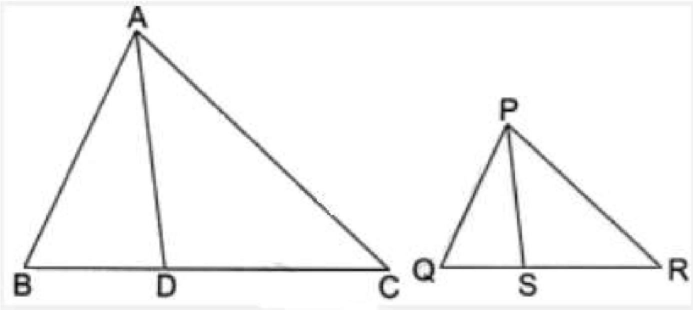
ΔABD ~ ΔPSR

ΔABC ~ ΔPSR

Solution

In the given diagram, ΔABC ~ ΔPQR. If AD and PS are bisectors of ∠BAC and ∠QPR respectively then ΔABD ~ ΔPQS.

Explanation:



Here, $\Delta ABC \sim \Delta PQR$

$\therefore \angle A = \angle P$

Then, $\frac{1}{2}\angle A = \frac{1}{2}\angle P$ or $\angle BAD = \angle QPS$ ...(i)

And $\angle B = \angle Q$ ...(ii)

In ΔABD and ΔPQS,

$\angle BAD = \angle QPS$ ...[From (i)]

$\angle B = \angle Q$ ...[From (ii)]

Then, $\Delta ABD \sim \Delta PQS$ ...(By AA similarity criterion)

Question 2.

2.1.

A = $\begin{bmatrix} x & 0 \\ 1 & 1 \end{bmatrix}$, B = $\begin{bmatrix} 4 & 0 \\ y & 1 \end{bmatrix}$ and C = $\begin{bmatrix} 4 & 0 \\ x & 1 \end{bmatrix}$. Find the values of x and y, if AB = C.

Solution

Given A = $\begin{bmatrix} x & 0 \\ 1 & 1 \end{bmatrix}$, B = $\begin{bmatrix} 4 & 0 \\ y & 1 \end{bmatrix}$, C = $\begin{bmatrix} 4 & 0 \\ x & 1 \end{bmatrix}$

Now, AB = C

$$\begin{bmatrix} x & 0 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 4 & 0 \\ y & 1 \end{bmatrix} = \begin{bmatrix} 4 & 0 \\ x & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 4x & 0 \\ 4+y & 1 \end{bmatrix} = \begin{bmatrix} 4 & 0 \\ x & 1 \end{bmatrix}$$

Then, by equality of matrix

$\therefore 4x = 4$

$\Rightarrow x = 1$

And $4 + y = x$

$\Rightarrow 4 + y = 1$

$y = -3$

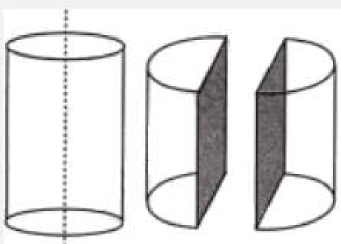
Hence, $x = 1$ and $y = -3$.

2.2. A solid metallic cylinder is cut into two identical halves along its height (as shown in the diagram). The diameter of the cylinder is 7 cm and the height is 10 cm.

Find:

a. The total surface area (both the halves).

b. The total cost of painting the two halves at the rate of ₹ 30 per cm^2 $\left(\text{Use } \pi = \frac{22}{7}\right)$



Solution

Here, radius of cylinder (r) = $\frac{7}{2}$ cm ...(∴ d = 7 cm)

Height of cylinder = 10 cm

a. T.S.A of a half cylinder

$$\frac{\pi r^2}{2} + \frac{\pi r^2}{2} + \frac{2\pi r h}{2} + d \times h$$

$$= \pi r^2 + \pi r h + d \times h$$

$$= \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} + \frac{22}{7} \times \frac{7}{2} \times 10 + 7 \times 10$$

$$= \frac{77}{2} + 110 + 70$$

$$= \frac{77}{2} + 180$$

$$= \frac{77 + 360}{2}$$

$$= \frac{437}{2}$$

= 218.5 cm2

So, total surface area of each half is 218.5 cm2.

b. Cost of painting = Total surface area × Rate of painting

= (218.5 + 218.5) × 30

= ₹ 13,110

2.3. 15, 30, 60, 120.... are in G.P. (Geometric Progression):

a) Find the nth term of this G.P. in terms of n.

b) How many terms of the above G.P. will give the sum 945?

Solution

a. Given, G.P. is 15, 30, 60, 120....

Here, a = 15

Common ratio (r) = $\frac{30}{15}$ = 2

Then $a_n = ar^{n-1}$

$= 15(2)^{n-1}$

b. Sum of n terms,

$$S_n = \frac{a(r^n - 1)}{r - 1} \quad ...(\because r > 1)$$

$$\Rightarrow 945 = 15\frac{(2^n - 1)}{2 - 1}$$

$$\Rightarrow \frac{945}{15} = 2^n - 1$$

$\Rightarrow 63 = 2^n - 1$

$\Rightarrow 2^n = 64$

$\Rightarrow 2^n = 2^6$

$\therefore n = 6$

Hence, number of terms needed are 6.

Question 3.

3.1. Factorize: $\sin^3\theta + \cos^3\theta$

Hence, prove the following identity:

$$\frac{\sin^3\theta + \cos^3\theta}{\sin\theta + \cos\theta} + \sin\theta\cos\theta = 1$$

Solution

$\sin^3\theta + \cos^3\theta$

$=(\sin\theta + \cos\theta)(\sin^2\theta + \cos^2 - \sin\theta\cos\theta)$

$= (\sin\theta + \cos\theta)(1 - \sin\theta\cos\theta)$. ...(i)

$$\text{L.H.S} = \frac{\sin^3\theta + \cos^3\theta}{\sin\theta + \cos\theta} + \sin\theta\cos\theta$$

$$= \frac{(\sin\theta + \cos\theta)(1 - \sin\theta\cos\theta)}{(\sin\theta + \cos\theta)} + \sin\theta\cos\theta \quad \text{...(From(i))}$$

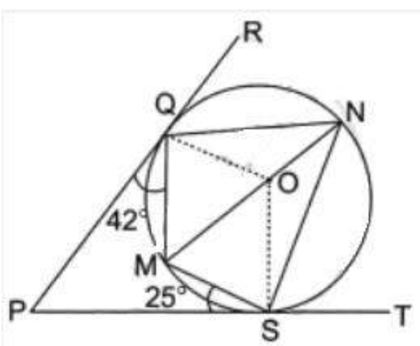
$= 1 - \sin\theta\cos\theta + \sin\theta + \cos\theta$

$= 1$

= R.H.S.

3.2. In the given diagram, O is the centre of the circle. PR and PT are two tangents drawn from the external point P and touching the circle at Q and S respectively. MN is a diameter of the circle. Given $\angle PQM = 42°$ and $\angle PSM = 25°$.

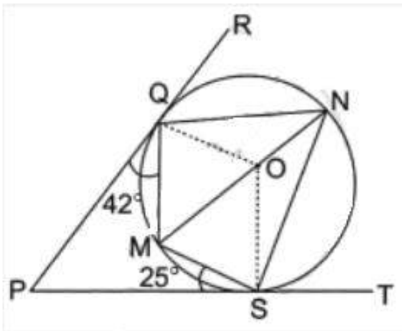
Find:



a) <OQM
b) <QNS
c) <QOS
d) <QMS

Solution

a. PR and PT are tangents to the circle with centre O.



Then, <OQP = 90°
As, radius is ⊥ to the tangent

Then, <OQM = <OQP – <MQP
= 90° – 42°
= 48°
b. <PQM = <QNM = 42° ...(By alternate segment theorem)

<PSM = <SNM = 25°

Then <QNS = <QNM + <SNM
= 42° + 25°
= 67°

c. <QOS = 2<QNS ...(since, angle subtended by the arc at the centre is twice the angle subtended by the arc at any other point of the circles.)

= 2 × 67°

= 134°

d. QNSN is a cyclic quadritateral
<QNS + <QMS = 180°

<QMS = 180° – 67° = 113°

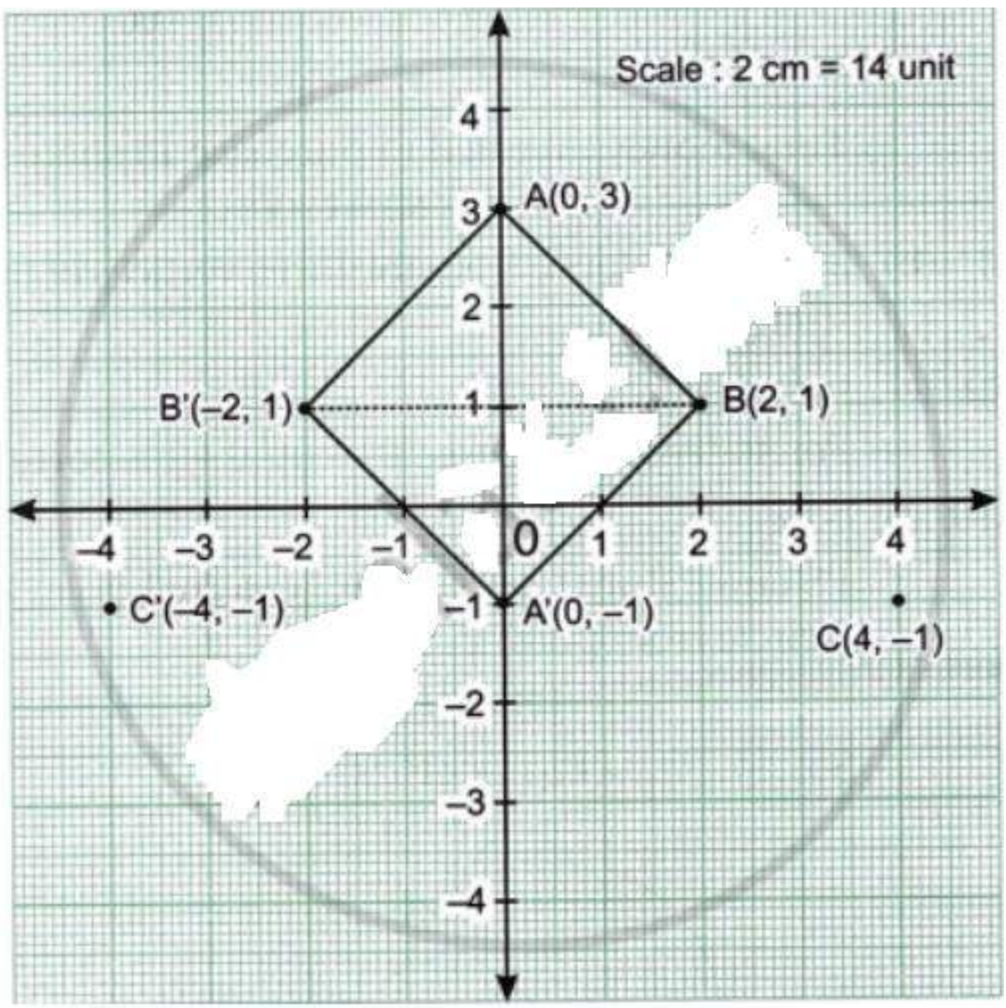
3.3. Use graph sheet for this question. Take 2 cm = 1 unit along the axes.

a. Plot A(0, 3), B(2, 1) and C(4, –1).

b. Reflect point B and C in y-axis and name their images as B' and C' respectively. Plot and write coordinates of the points B' and C'.
c. Reflect point A in the line BB' and name its images as A'.
d. Plot and write coordinates of point A'.
e. Join the points ABA'B' and give the geometrical name of the closed figure so formed.

Solution

a.



a. B'(– 2, 1), C'(–4, –1)

b. A'(0, –1)

c. Rhombus has a diagonal that bisects at 90° and all form sides are equal.

SECTION-B (40 Marks) (Attempt any four questions from this Section.)

Question 4.

4.1. Suresh has a recurring deposit account in a bank. He deposits ₹ 2000 per month and the bank pays interest at the rate of 8% per annum. If he gets ₹ 1040 as interest at the time of maturity, find in years total time for which the account was held.

Solution

Deposit per month P = ₹ 2000
Rate of interest R = 8%
Interest earned, I = ₹ 1040
Let n months be the length of time for which money is invested.
Then, by formula

$$I = \frac{P \times n(n+1)}{2 \times 12} \times \frac{r}{100}$$

$$1040 = 2000 \times \frac{n(n+1)}{2 \times 12} \times \frac{8}{100}$$

$$1040 = \frac{20 \times n \times (n+1)}{3}$$

$52 \times 3 = n^2 + n$

$n^2 + n - 156 = 0$

$n^2 + 13n - 12n - 156 = 0$

$n(n + 13) - 12(n + 13) = 0$

$(n - 12)(n + 13) = 0$

$n = 12$ ...($\because$ $n = -13$, is not possible)

As a result, the time period for which money is invested is 12 months or one year.

4.2. The following table gives the duration of movies in minutes:

Duration	100 – 110	110 – 120	120 – 130	130 – 140	140 – 150	150 – 160

No. of movies	5	10	17	8	6	4

Using step-deviation method, find the mean duration of the movies.

Solution

Duration (in minutes)	No. of movies fi	xi	$u_i = \frac{x_i - A}{h}$	fiui
100 – 110	5	105	–3	–15
110 – 120	10	115	–2	–20
120 – 130	17	125	–1	–17
130 – 140	8	135 = A	0	0
140 – 150	6	145	1	6
150 – 160	4	155	2	5
Total	**50**			**–38**

$$\overline{x} = \frac{\sum f_i u_i}{\sum f} \times h$$

$$= 135 + \frac{(-38)}{50} \times 10$$

= 135 – 7.6

= 127.4

4.3.

If $\frac{(a+b)^3}{(a-b)^3} = \frac{64}{27}$.

a. Find $\frac{a+b}{a-b}$

b. Hence using properties of proportion, find a : b.

Solution

a. Given $\frac{(a+b)^3}{(a-b)^3} = \frac{64}{27}$

Taking cube root on both sides, we get

$$\frac{a+b}{a-b} = \sqrt[3]{\frac{64}{27}}$$

$$\Rightarrow \frac{a+b}{a-b} = \frac{4}{3}$$

b. Now $\frac{a+b}{a-b} = \frac{4}{3}$

Applying componendo and dividendo, we get

$$\frac{(a+b)+(a-b)}{(a+b)-(a-b)} = \frac{4+3}{4-3}$$

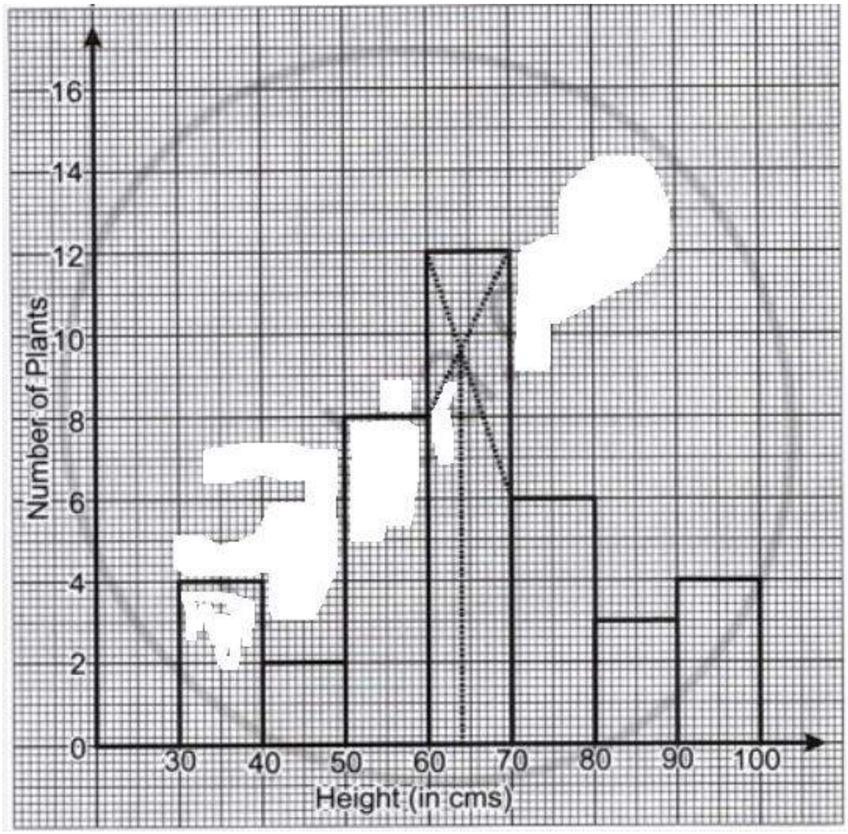
$$\Rightarrow \frac{2a}{2b} = \frac{7}{1}$$

$$\Rightarrow \frac{a}{b} = \frac{7}{1}$$

Hence, a : b = 7 : 1

Question 5.

5.1. The given graph with a histogram represents the number of plants of different heights grown in a school campus. Study the graph carefully and answer the following questions:



a. Make a frequency table with respect to the class boundaries and their corresponding frequencies.
b. State the modal class.
c. Identify and note down the mode of the distribution.
d. Find the number of plants whose height range is between 80 cm to 90 cm.

Solution

a.

CI.	Frequency
30 – 40	4
40 – 50	2
50 – 60	8
60 – 70	12
70 – 80	6

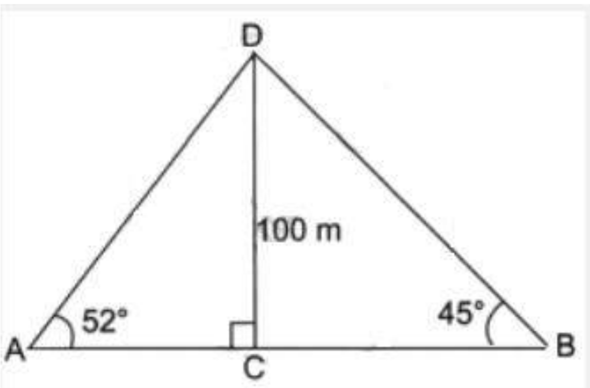
80 – 90	3
90 – 100	4

b. Here, modal class is 60 – 70, with highest frequency of 12.

c. From the given graph the mode of the distribution is 64.

d. The number of plants whose height range is between 80 cm to 90 cm is 3.

5.2. The angle of elevation of the top of a 100 m high tree from two points A and B on the opposite side of the tree are 52° and 45° respectively. Find the distance AB, to the nearest metre.



Solution

In ΔADC,

$\tan 52° = \frac{DC}{AC} = \frac{100}{AC}$

$\Rightarrow 1.2799 = \frac{100}{AC}$ ...(From table)

$\Rightarrow AC = \frac{100}{1.2799}$

$\Rightarrow AC = 78.13$ m

In ΔBCD,

$\tan 45° = \frac{CD}{BC}$

$\Rightarrow 1 = \frac{100}{BC}$

BC = 100 m

∴ AB = AC + BC

= 78.13 + 100

= 178.13 m

Hence, the distance AB is 178 m ...(approx)

Question 6.

6.1. Solve the following equation for x and give, in the following case, your answer correct to 2 decimal places:

$2x^2 - 10x + 5 = 0$

Solution

Given, 2x2 – 10x + 5 = 0

On comparing it with the equation ax2 + bx + c = 0, we get,

a = 2, b = – 10, c = 5

By using formula,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= -\frac{(-10) \pm \sqrt{(-10)^2 - 4 \times 2 \times 5}}{2 \times 2}$$

$$= \frac{10 \pm \sqrt{100 - 40}}{4}$$

$$= \frac{10 \pm 2\sqrt{15}}{4}$$

$$= \frac{10 \pm 2 \times 3.873}{4}$$

$$= \frac{10 \pm 7.758}{4}$$

Then, $= \frac{10 + 7.758}{4}$ and $\frac{10 - 7.758}{4}$

$= \frac{17.758}{4}$ and $\frac{2.242}{4}$

= 4.4395 and 0.5605

Hence, x = 4.440 and 0.561

6.2. The nth term of an Arithmetic Progression (A.P.) is given by the relation Tn = 6(7 – n)..

Find:

a. its first term and common difference
b. sum of its first 25 terms

Solution

Given, Tn = 6(7 – n)

a. For first term, put n = 1

Then, a1 = 6(7 – 1)

= 6 × 6

= 36

For second term, put n = 2

Then a2 = 6(7 – 2)

= 6 × 5

= 30

Then, common difference

∴ d = a2 – a1

= 30 – 36

= – 6

Hence, first term is 36 and common difference is - 6.

b. $S_n = \frac{n}{2}[2a + (n-1)d]$

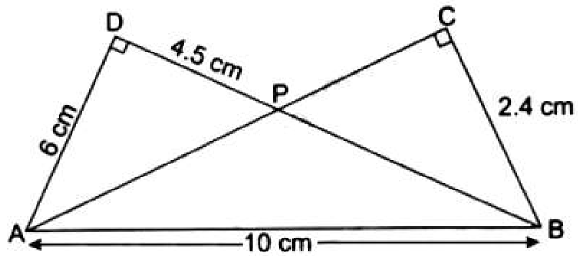
$S_{25} = \frac{25}{2}[2 \times 36 + (25-1)(-6)]$

= $\frac{25}{2}[72 - 144]$

= $\frac{25}{2} \times (-72)$

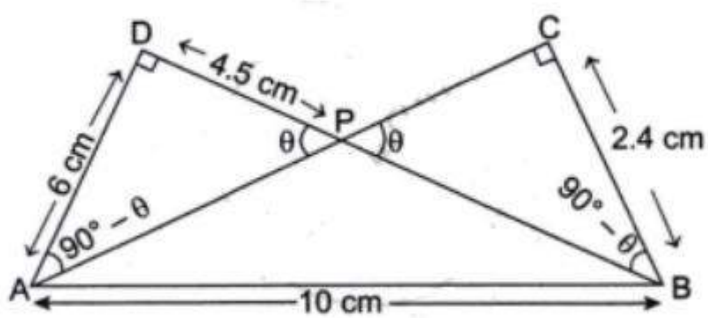
S_{25} = – 900

6.3. In the given diagram ΔADB and ΔACB are two right angled triangles with ∠ADB = ∠BCA = 90°. If AB = 10 cm, AD = 6 cm, BC = 2.4 cm and DP = 4.5 cm.



a. Prove that ΔAPD ~ ΔBPC
b. Find the length of BD and PB
c. Hence, find the length of PA
d. Find area ΔAPD : area ΔBPC.

Solution



Given: In ΔADB and ΔACB,

∠ADB = ∠BCA = 90°

AB = 10 cm, AD = 6 cm, BC = 2.4 cm, DP = 4.5 cm

a. In ΔAPD and ΔBPC

∠APD = ∠BPC ...(Vertically opposite angles)

∠ADP = ∠BCP = 90°

∴ ΔAPD ~ ΔBCP ...(By AA similarity criterion)

b. In ΔABD,

By pythagoras theorem,

AB2 = AD2 + BD2

(10)2 = 62 + (BD)2

BD2 = 100 – 36 = 64

BD = 8 cm

Then, PB = BD – PD

= 8 – 4.5

= 3.5 cm

c. In ΔPAD,

By pythagoras theorem,

AP2 = AD2 + PD2

AP2 = 62 + (4.5)2

= 36 + 20.25

= 56.25

AP = $\sqrt{56.25}$ cm

AP = 7.5 cm

d. Since, ΔAPD ~ ΔBPC

$$\therefore \frac{ar(\Delta APD)}{ar(\Delta BPC)} = \frac{AD^2}{BC^2}$$

$$= \frac{6 \times 6}{2.4 \times 2.4}$$

$$= \frac{1 \times 1}{0.4 \times 0.4}$$

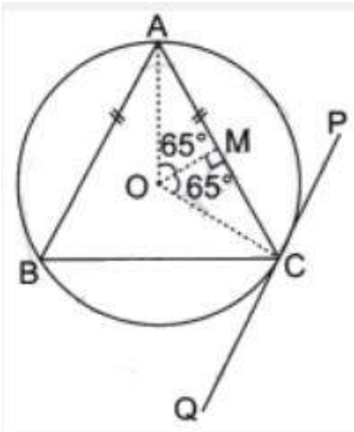
$$= \frac{10 \times 10}{4 \times 4}$$

$$= \frac{25}{4}$$

Hence, ar(ΔAPD) : ar(ΔBPC) = 25 : 4

Question 7.

7.1. In the given diagram an isosceles ΔABC is inscribed in a circle with centre O. PQ is a tangent to the circle at C. OM is perpendicular to chord AC and ∠COM = 65°.



Find:

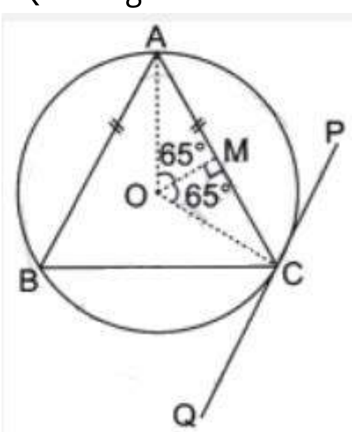
a. ∠ABC

b. ∠BAC

c. ∠BCQ

Solution

PQ is tangent to circle OM is perpendicular PQ chord AC and <COM = 65°



a. Here, ∠AOM = ∠COM = 65°

= 65° + 65°

= 130°

Now, ∠ABC = $\frac{1}{2}$ ∠AOC ...(Since, angle at the centre is twice the angle formed by the same arc at any other point of the circle)

= $\frac{1}{2} \times 130^\circ$

b. In ΔABC,

AB = AC

<ABC = <ACB = 65° ...(Since, angles opposite to equal sides are equal)

∴ ∠BAC = 180° – (65° + 65°)

= 180° – 130°

= 50°

c. <OCQ = 90° ...(Since, angle between the radius and the tangent is 90°)

In ΔOMC,

<OCM = 180° – (<OMC + <MOC) ...[By angle sum property of triangle]

= 180° – (90° + 65°)

= 180° – 155°

= 25°

<ACB = 65°

<OCB = <ACB – <OCM

= 65° – 25°

= 40°

<BCQ = <OCQ – ∠OCB

= 90° – 40°

= 50°

7.2. Solve the following inequation, write down the solution set and represent it on the real number line.

$$-3+x \le \frac{7x}{2}+2 < 8+2x, x \in I$$

Solution

Given: $-3+x \le \frac{7x}{2}+2 < 8+2x, x \in I$

Then, $-3+x \le \frac{7x}{2}+2$

$\Rightarrow -3-2 \le \frac{7x}{2}-x$

$\Rightarrow -5 \le \frac{7x-2x}{2}$

$\Rightarrow -10 \le 5x$

$\Rightarrow -2 \le x$ or $x \ge -2$

And $\frac{7x}{2}+2 < 8+2x$

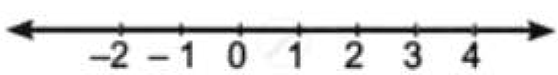
$\Rightarrow \frac{7x}{2}-2x < 8-2$

$\Rightarrow \frac{7x-4x}{2} < 6$

$\Rightarrow 3x < 12$

$\Rightarrow x < 4$

$\Rightarrow -2 \le x < 4$



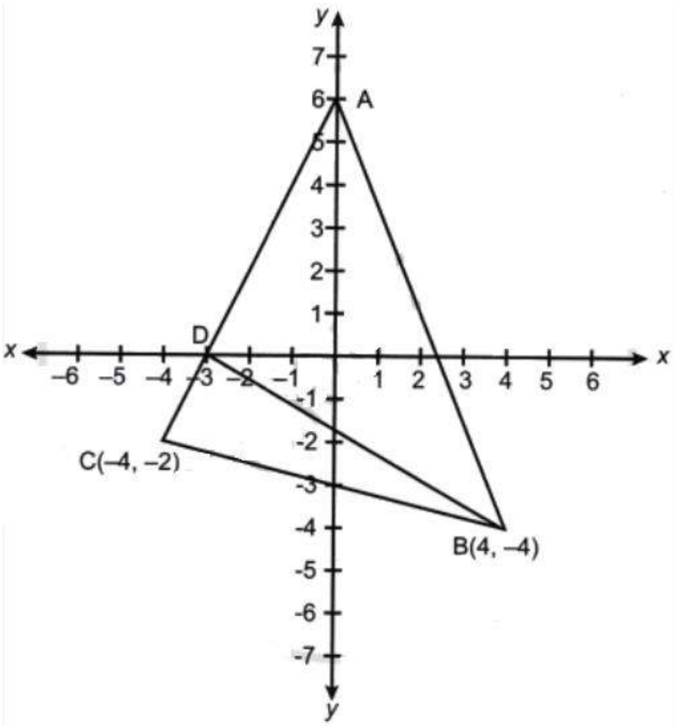
7.3. In the given diagram, ABC is a triangle, where B(4, – 4) and C(– 4, –2). D is a point on AC.

a. Write down the coordinates of A and D.

b. Find the coordinates of the centroid of ΔABC.

c. If D divides AC in the ratio k : 1, find the value of k.

d. Find the equation of the line BD.



Solution

a. Coordinates of A = (0, 6)
   Coordinates of D = (–3, 0)

b. Here, coordinates of A = (0, 6)
   Coordinates of B = (4, –4)
   Coordinates of C = (–4, –2)
   Then, coordinates of centroid

$$\left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right)$$

$$= \left(\frac{0 + 4 + (-4)}{3}, \frac{6 + (-4) + (-2)}{3}\right)$$

$$= \left(\frac{0}{3}, \frac{0}{3}\right)$$

$$= (0, 0)$$

c. Here, $x_1 = -4$, $y_1 = -2$

$x_2 = 0$, $y_2 = 6$

$m_1 = k$, $m_2 = 1$

$x = -3$, $y = 0$

By section formula,

$$D(x, y) = \left(\frac{m_1x_2 + m_2x_1}{m_1 + m_2}, \frac{m_1y_2 + m_2y_1}{m_1 + m_2}\right)$$

$$D(-3, 0) = \left(\frac{k \times 0 + 1 \times (-4)}{k+1}, \frac{k \times 6 + 1 \times (-2)}{k+1}\right)$$

$$\therefore -3 = \frac{-4}{k+1} \text{ or } 0 = \frac{6k-2}{-k+1}$$

$\Rightarrow -3k - 3 = -4$ or $6k - 2 = 0$

$\Rightarrow -3k = -1$ or $6k = 2$

$$k = \frac{1}{3} \text{ or } k = \frac{1}{3}$$

$$\text{Hence, } k = \frac{1}{3}$$

d. Coordinates of B = (4, – 4)

Coordinates of D = (–3, 0)

Then, equation of line BD is:

$$(y - y_1) = \frac{y_2 - y_1}{(x_2 - x_1)}(x - x_1)$$

$$\Rightarrow [y - (-4)] = \frac{[0 - (-4)]}{(-3 - 4)}(x - 4)$$

$$\Rightarrow (y + 4) = \frac{4}{-7}(x - 4)$$

$\Rightarrow -7(y + 4) = 4(x - 4)$

$\Rightarrow -7y - 28 = 4x - 16$

$\Rightarrow 4x - 16 + 7y + 28 = 0$

$\Rightarrow 4x + y + 12 = 0$, is the required equation

Question 8.

8.1. The polynomial $3x^3 + 8x^2 - 15x + k$ has $(x - 1)$ as a factor. Find the value of k. Hence factorize the resulting polynomial completely.

Solution

Given, $P(x) = 3x^3 + 8x^2 - 15x + k$

Put $x - 1 = 0$

$x = 1$

Now, $P(1) = 3(1)^3 + 8(1)^2 - 15(1) + k = 0$

$\Rightarrow 3 + 8 - 15 + k = 0$

$\Rightarrow -4 + k = 0$

$\Rightarrow k = 4$

Hence, $k = 4$

Factorization:

$P(x) = 3x^3 + 8x^2 - 15x + 4$

$$x - 1)\overline{3x^3 + 8x^2 - 15x + 4}\,(3x^2 + 11x - 4$$

$$\begin{array}{l}
3x^3 - 3x^2 \\
\underline{-\quad +\qquad\qquad} \\
\quad 11x^2 - 15x \\
\quad 11x^2 - 11x \\
\underline{\quad -\quad +\qquad\qquad} \\
\qquad -4x + 4 \\
\qquad -4x + 4 \\
\underline{\qquad +\quad -\qquad}
\end{array}$$

$\therefore 3x^3 + 8x^2 - 15x + 4 = (x - 1)(3x^2 + 11x - 4)$

$= (x - 1)(3x^2 + 12x - x - 4)$

$= (x - 1)[3x(x + 4) - 1(x + 4)]$

$= (x - 1)(3x - 1)(x + 4)$

8.2. The following letters A, D, M, N, O, S, U, Y of the English alphabet are written on separate cards and put in a box. The cards are well shuffled and one card is drawn at random. What is the probability that the card drawn is a letter of the word,

a. MONDAY?

b. Which does not appear in MONDAY?

c. Which appears both in SUNDAY and MONDAY?

Solution

Total outcomes n(s) = 8

a. Favourable outcomes, n(E) = 6

$$P(E) = \frac{n(E)}{n(S)}$$

$$= \frac{6}{8}$$

$$= \frac{3}{4}$$

b. $P\left(\overline{E}\right) = 1 - P(E)$

$$= 1 - \frac{3}{4}$$

$$= \frac{4-3}{4}$$

$$= \frac{1}{4}$$

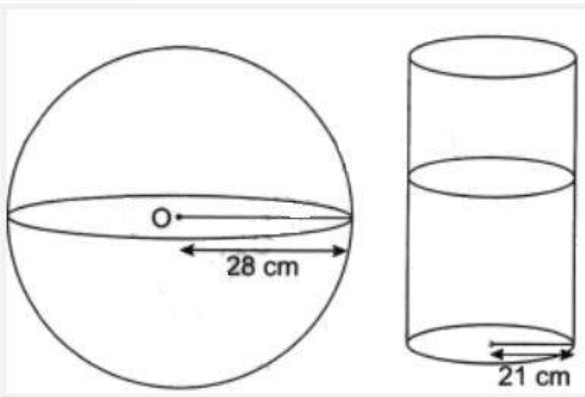
c. Favourable outcomes = uncommon letters

= N, D, A, Y

= 4

Then, required probability = $\frac{4}{8} = \frac{1}{2}$

8.3. Oil is stored in a spherical vessel occupying $\frac{3}{4}$ of its full capacity. Radius of this spherical vessel is 28 cm. This oil is then poured into a cylindrical vessel with a radius of 21 cm. Find the height of the oil in the cylindrical vessel (correct to the nearest cm). Take $\pi = \frac{22}{7}$



Solution

Radius of spherical vessel, R = 28 cm

Radius of cylindrical vessel, r = 21 cm

Let, the height of cylindrical vessel be h cm

Volume of oil in sphere = $\frac{3}{4} \times \frac{4}{3}\pi R^3$

$= \frac{22}{7} \times 28 \times 28 \times 28$

Then, volume of oil in cylindrical vessel = Volume of oil in spherical vessel

$$\Rightarrow \pi \times r^2 h = \frac{22}{7} \times 28 \times 28 \times 28$$

$$\Rightarrow \frac{22}{7} \times 21 \times 21 \times h = \frac{22}{7} \times 28 \times 28 \times 28$$

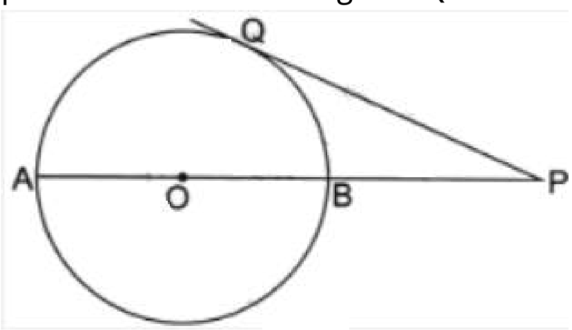
$$\Rightarrow h = \frac{28 \times 28 \times 28}{21 \times 21}$$

$$= \frac{4 \times 4 \times 28}{3 \times 3}$$

= 49.78 cm

Question 9.

9.1. The figure shows a circle of radius 9 cm with 0 as the centre. The diameter AB produced meets the tangent PQ at P. If PA = 24 cm, find the length of tangent PQ:



Solution

Given, Radius of circle (r), OA = OB = 9 cm
Here, PA = 24 cm
Then PB = PA – AB
= 24 – 18
= 6 cm
Then, PB × PA = PQ^2 ...(By property)
⇒ 6 × 24 = PQ^2
⇒ PQ = $\sqrt{2 \times 2 \times 2 \times 2 \times 3 \times 3}$
= 2 × 2 × 3
= 12 cm
Hence, the length of tangent PQ is 12 cm.

9.2. Mr. Gupta invested ₹ 33000 in buying ₹ 100 shares of a company at 10% premium. The dividend declared by the company is 12%.

Find:

a. the number of shares purchased by him

b. his annual dividend.

Solution

Money invested = ₹ 3,000
N.V. = ₹ 100

$$\text{M.V.} = ₹\left(100 + \frac{10}{100} \times 100\right) \times ₹\,100$$

Dividend given = 12%

a. Number of shares purchased = $\frac{33,000}{110} = 300$

b. Annual dividend = Number of shares × Rate of dividend × Face value of one share

$$= 300 \times \frac{12}{100} \times 100$$

= ₹ 3600

9.3. A life insurance agent found the following data for distribution of ages of 100 policy holders.

Age in years	**Policy Holders (frequency)**	**Cumulative frequency**
20 – 25	2	2
25 – 30	4	6
30 – 35	12	18
35 – 40	20	38
40 – 45	28	66

45 – 50	22	88
50 – 55	8	96
55 – 60	4	100

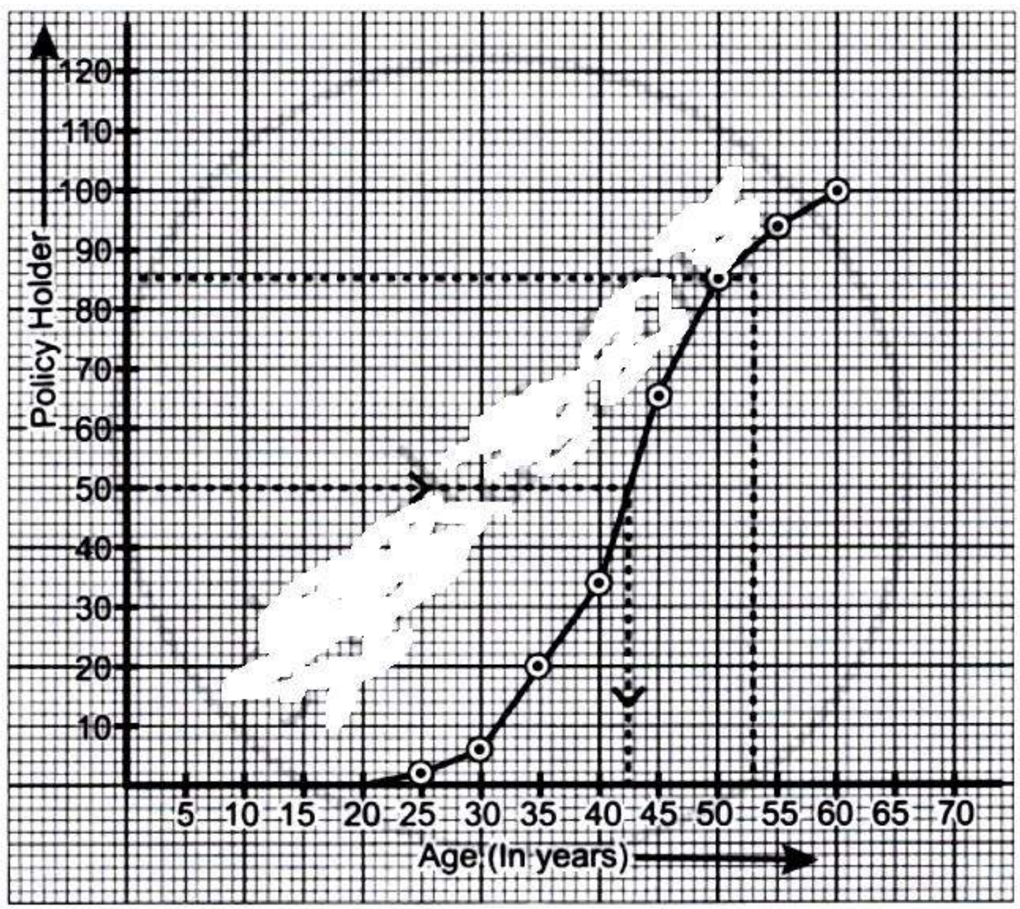
On a graph sheet draw an ogive using the given data. Take 2 cm = 5 years along one axis and 2 cm = 10 policy holders along the other axis.

Use your graph to find:

a. The median age.

b. Number of policy holders whose age is above 52 years.

Solution



Here, N = 100

Then, $\frac{N}{2} = \frac{100}{2} = 50$

a. Median age = 43 years

b. Number of policy holders who are 52 years old = 85

∴ Required number of policy holders = 100 – 85 = 15

Question 10.

10.1. Rohan bought the following eatables for his friends:

Soham Sweet Mart: Bill				
S.N.	**Item**	**Price**	**Quantity**	**Rate of GST**
1	Laddu	₹ 500 per kg	2 kg	5%
2	Pastries	₹ 100 per kg	12 pieces	18%

Calculate:

a. Total GST paid.

b. Total bill amount including GST.

Solution

Soham Sweet Mart: Bill						
S.N.	**Item**	**Price**	**Quantity**	**Rate of GST**	**Total Price**	**GST**
1	Laddu	₹ 500 per kg	2 kg	5%	₹ 1000	₹ 50
2	Pastries	₹ 100 per kg	12 pieces	18%	₹ 1200	₹ 216

Total GST paid
= 50 + 216
= ₹ 266

Total bill including GST
= ₹ 1000 + ₹ 50 + ₹ 1200 + ₹ 216
= ₹ 2,466

10.2.

10.2.a

If the lines kx – y + 4 = 0 and 2y = 6x + 7 are perpendicular to each other, find the value of k.

Solution

Given lines are

kx – y + 4 = 0

And 2y = 6x + 7

Or y = kx + 4 ...(i)

And y = $3x + \frac{7}{2}$ ...(ii)

On comparing with y = m_x + c, we get

m_1 = k and m_2 = 3

If lines are perpendicular, then

$m_1m_2 = -1$

⇒ k × 3 = – 1

⇒ k = $\frac{-1}{3}$

10.2.b

Find the equation of a line parallel to 2y = 6x + 7 and passing through (–1, 1).

Solution

Given the equation of a line,

2y = 6x + 7

Or y = $3x + \frac{7}{2}$

Here, m = 3

The equation of a line with slope, m = 3 and passing through (–1, 1) is:

$(y - y_1) = m(x - x_1)$

$\Rightarrow (y - 1) = m(x + 1)$

$\Rightarrow y - 1 = 3x + 3$

$\Rightarrow y = 3x + 3 + 1$

$\Rightarrow y = 3x + 4$, is the required equation

10.3.

Use ruler and compass to answer this question. Construct ∠ABC = 90°, where AB = 6 cm, BC = 8 cm.

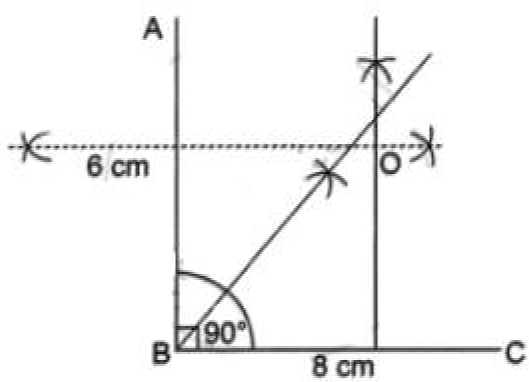
a. Construct the locus of points equidistant from B and C.

b. Construct the locus of points equidistant from A and B.

c. Mark the point which satisfies both the conditions (a) and (b) as 0. Construct the locus of points keeping a fixed distance OA from the fixed point 0.

d. Construct the locus of points which are equidistant from BA and BC.

Solution



a. The locus of points equidistant from B and C is on BC's perpendicular bisector.

b. Similarly, the locus will be at the perpendicular bisector of AB.

c. The locus will be the circle that touches all three points A, B and C.

d. The point equidistant from BA and BC will be the angle bisector of ∠ABC.